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Superfield formulation of OSp(1, 4) supersymmetry

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Abstract. A self-contained superfield approach to global supersymmetry in anti-de Sitter space (OSp(1,4)) is developed. General transformation laws for OSp(1,4) superfields are established, and all basic elements of the OSp(1,4)-covariant formalism, such as covariant superfield derivatives, invariant integration measures over the superspace OSp(1,4)/O(1,3) in both the real and shifted bases, relations between different parametrisations of superspace, etc., are given explicitly. We also analyse the reducibility questions, focusing in particular on the structure of chiral representations of OSp(1,4). The simplest linear OSp(1,4)-invariant models are constructed: the OSp(1,4) analogue of the Wess–Zumino model and OSp(1,4) extension of the Yang–Mills theory. The first model, together with the spontaneous breaking of OSp(1,4), exhibits an effect of the spontaneous violation of P and CP parities with the strength related to the anti-de Sitter radius. We discuss the relation of the proposed approach to supergravity.

1. Introduction

At the present time a good deal of attention is being paid to the orthosymplectic supergroup OSp(1,4), the minimal extension of the group O(2,3) (~ Sp(4)) by Majorana spinor generators (Keck 1975, Deser and Zumino 1977, Zumino 1977, MacDowell and Mansouri 1977, Mansouri 1977, Chamseddine 1977, 1978, Gürsey and Marchildon 1978a, b, Ivanov and Sorin 1979a,b).

Obvious indications that this supergroup has strong relevance to the dynamics of supersymmetric theories exist. For instance, Deser and Zumino (1977) have argued that spontaneously broken supergravity should be constructed as a theory of the spontaneously broken local OSp(1,4) symmetry (OSp(N,4) for the O(N) extended supergravity). With such a construction, it becomes possible to remove the unwanted cosmological term arising due to the super-Higgs effect (through cancellation with a similar term coming from the pure gauge supergravity Lagrangian) and, simultaneously, to adjust the reasonable order of the mass splitting between bosons and fermions. An analogous approach to spontaneously broken supergravity was developed on the basis of the vierbein formulation of OSp(1,4)symmetry (MacDowell and Mansouri 1977, Mansouri 1977, Chamseddine 1977, 1978, Gürsey and Marchildon 1978a, b).

Additional evidence in favour of the significance of OSp(1,4) is associated with its role as a subgroup in the Wess–Zumino (Wess and Zumino 1974a, b) conformal supergroup. In fact, the conformal supergroup is a closure of two of its different graded subgroups OSp(1,4) with the common O(2,3) subgroup (Ivanov and Sorin 1979a). We

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have shown (in the same paper) that one of these OSp(1,4) groups is the stability group of classical instanton-like solutions of the simplest superconformal-invariant theory, the massless Wess–Zumino model, i.e. it plays there the same role as the group O(2,3) in the massless ϕ^4 theory (Fubini 1976). The other OSp(1,4) is spontaneously broken on these solutions to O(2,3) symmetry. By analogy, the OSp(1,4) structure of spontaneously broken supergravity may be thought to emerge via a similar mechanism (Ivanov and Sorin 1979a,b). Note also that the Euclidean analogue of OSp(1,4), the extension of the group O(5) by Dirac spinor generators, may happen to be the stability group of generalised bosonic–fermionic instantons in Euclidean supersymmetric gauge theories (like O(5) in the usual Yang–Mills case (Jackiw and Rebbi 1976)).

Taking all of this into account, it seems of real importance to construct and analyse theories with global OSp(1,4) invariance, i.e. supersymmetric theories in anti-de Sitter space ~O(2,3)/O(1,3). The first non-trivial theory of this type, nonlinear realisation of the OSp(1,4) symmetry, has recently been considered by Zumino (1977). He has found, in particular, that the relevant Goldstone fermion possesses a mass which is twice the inverse radius of anti-de Sitter space. This result was reproduced in another context in our paper (Ivanov and Sorin 1979b) where the ordinary massless Wess–Zumino model was recognised as the simplest linear σ model of spontaneously broken conformal and OSp(1,4) supersymmetries. In the same paper, we constructed the OSp(1,4) analogue of the massive Wess–Zumino model and studied its vacuum structure. However, the methods we used to obtain the corresponding Lagrangians were, to a great extent, heuristic. It is desirable to have general algorithms for constructing models with a linear realisation of OSp(1,4) analogous to those employed in the usual supersymmetry.

The most adequate and elegant formulation of conventional linear supersymmetric theories is achieved using the superfield concept (Salam and Strathdee 1975). The present paper is devoted to the description of a consistent superfield approach to OSp(1,4) supersymmetry.

The supergroup OSp(1,4) can be realised naturally in the superspace ~OSp(1,4)/O(1,3), the spinorial extension of anti-de Sitter space ~O(2,3)/O(1,3). The first to consider such a realisation was Keck (1975). He studied the transformation properties of a general scalar OSp(1,4) superfield and reduced it to irreducible pieces. However, it was still not clear how to construct OSp(1,4) invariants from superfields and hence how to set up non-trivial Lagrangian densities. We give explicitly all elements relevant to the construction of OSp(1,4)-invariant Lagrangians of arbitrary structure: covariant derivatives of superfields with any external Lorentz index, invariant measures of integration over superspace, relations between different parametrisations of superspace, etc.

The paper is organised as follows. In § 2, we describe the OSp(1,4)-covariant superfield technique in the symmetrically parametrised superspace by proceeding from the general theory of group realisations in homogeneous spaces. In § 3, we pass to the non-symmetric parametrisation and study representations of OSp(1,4) in the left- and right-handed chiral superspaces. Section 4 contains examples of simplest OSp(1,4)-invariant models constructed using the general recipes of previous sections. We present the OSp(1,4) analogue of the Wess–Zumino model and the OSp(1,4)-invariant extension of the Yang–Mills theory and examine their properties. In appendices 1, 2, 3 and 4 some technical questions are considered and a number of useful formulae are quoted. The relation with supergravity is briefly explained in the conclusion and, as short comments, throughout the text.

2. OSp(1,4) superfields in a symmetric basis

2.1. *The superalgebra OSp(1,4)*

The structure relations of OSp(1,4) can be taken as (Keck 1975, Zumino 1977)†

$$[M_{\mu\nu}, M_{\lambda\rho}] = \mathrm{i}(\eta_{\mu\rho}M_{\nu\lambda} + \eta_{\nu\lambda}M_{\mu\rho} - \eta_{\mu\lambda}M_{\nu\rho} - \eta_{\nu\rho}M_{\mu\lambda}),$$
$$[M_{\mu\nu}, R_\lambda] = \mathrm{i}(\eta_{\nu\lambda}R_\mu - \eta_{\mu\lambda}R_\nu] \qquad [R_\mu, R_\nu] = -\mathrm{i}m^2 M_{\mu\nu}, \tag{2.1a}$$
$$[M_{\mu\nu}, Q] = -\tfrac{1}{2}\sigma_{\mu\nu}Q, \qquad [R_\mu, Q] = -\tfrac{1}{2}m\gamma_\mu Q,$$
$$\{Q, \bar{Q}\} = \gamma_\mu R^\mu + \tfrac{1}{2}m\sigma^{\mu\nu}M_{\mu\nu}. \tag{2.1b}$$

Even generators R_μ and $M_{\mu\nu}$ form the algebra of the group O(2,3), $M_{\mu\nu}$ being the Lorentz subgroup generators. The odd generator Q has the transformation properties of the O(2,3) spinor and obeys the Majorana condition $Q = C\bar{Q}^{\mathrm{T}}$. We have introduced explicitly into (2.1) the dimensional parameter of contraction $m([m] = L^{-1})$ to have at each step a clear correspondence with the standard supersymmetry whose algebra results from (2.1) in the limit $m \to 0$.

In what follows (§ 3) we shall need a more detailed knowledge of the structure of the superalgebra (2.1). It can be represented as a closure of two complex-conjugated superalgebras $S_\pm \propto (M_{\mu\nu}, Q_\pm = \frac{1}{2}(1 \pm \mathrm{i}\gamma_5)Q)$ (we call them, respectively, the left- and right-handed Lorentz superalgebras):

$$[M_{\mu\nu}, Q_\pm] = -\tfrac{1}{2}\sigma_{\mu\nu}Q_\pm, \qquad \{Q_\pm, \bar{Q}_\pm\} = \tfrac{1}{2}m\sigma^{\mu\nu}\tfrac{1}{2}(1 \pm \mathrm{i}\gamma_5)M_{\mu\nu}, \tag{2.2}$$
$$\{Q_\pm, \bar{Q}_\mp\} = \gamma^\mu \tfrac{1}{2}(1 \mp \mathrm{i}\gamma_5)R_\mu. \tag{2.3}$$

In this sense OSp(1,4) resembles the conformal superalgebra, which is a closure of two of its supersubalgebras OSp(1,4) intersecting over the algebra of O(2,3) and shifted with respect to each other by the γ_5 rotation with angle π (Ivanov and Sorin 1979a). Note that OSp(1,4) is not the only possible closure of S_+ and S_-: taking the cross anticommutator of their spinor charges equal to zero one obtains a superalgebra OSp(1,2*c*), a minimal enlargement of the Lorentz group algebra by Majorana generators.

2.2. *Covariant methods in anti-de Sitter space*

Since O(2, 3) is the group of motions of anti-de Sitter space (Gürsey 1964, Hawking and Ellis 1973) its spinorial extension OSp(1,4) determines the simplest possible supersymmetry in this space. Like its Minkowski counterpart, OSp(1,4) symmetry admits a natural representation in a superspace x_μ, θ_α but with anti-de Sitter space as the even subspace (Keck 1975, Zumino 1977). Clearly, to construct the OSp(1,4)-covariant formalism in superspace one needs, first of all, the basic relations of the O(2,3)-covariant formalism in anti-de Sitter space. To our knowledge, the systematic derivation of these relations from a uniform group-theoretic point of view has not yet been given in the literature. For this reason, and also bearing in mind that the superspace case will be treated in an analogous way, we find it useful to present such a derivation in detail here.

† Our conventions on the metric and the γ matrices coincide with those of Salam and Strathdee (1975). Indices μ, ν, ρ, λ refer to Lorentz vectors and α, β, γ, δ to spinors. Summation over repeated indices is everywhere implied.

Anti-de Sitter space is a space of constant negative curvature homeomorphic to the homogeneous (coset) space O(2,3)/O(1,3). Because of this, we can take advantage of general constructive methods of group realisations in homogeneous spaces (see, e.g., Helgason (1962) and, in the particle physics context, Coleman *et al* (1969), Callan *et al* (1969), Volkov (1973) and Ogievetsky (1974)).

The one-to-one correspondence between the coset space O(2,3)/O(1,3) and anti-de Sitter space means that coordinates of the latter can be identified with parameters of left cosets of the group O(2,3) over its subgroup O(1,3). To different parametrisations of cosets there correspond equivalent systems of curvilinear coordinates in anti-de Sitter space. Of common use is the exponential parametrisation

$$\exp(\mathrm{i}z^{\mu}R_{\mu})L$$

where L is the set of the Lorentz group elements. However, we find it more convenient to deal with the coordinates x_{μ} related to z_{μ} by

$$x_{\mu}=z_{\mu}\frac{\tan\frac{1}{2}mz}{mz}, \qquad g(x)\overset{\mathrm{def}}{=}\exp(\mathrm{i}z^{\mu}(x)R_{\mu}) \tag{2.4}$$

with $z=(z^{\mu}z_{\mu})^{1/2}$.† This choice is advantageous as it diagonalises the metric of anti-de Sitter space and makes the appearance of the covariant derivatives of fields most simple. Note that the x_{μ} are stereographic projections of the Cartesian coordinates on a four-dimensional hypersphere with radius $1/m$ in a five-dimensional pseudo-Euclidean space with metric $(\eta_{\mu\nu}, 1)$‡.

The group O(2,3) can be realised in the space O(2,3)/O(1,3) as left multiplications of cosets:

$$g(x)\xrightarrow{g_0=\exp(\mathrm{i}\lambda^{\mu}R_{\mu}+\frac{1}{2}\mathrm{i}\lambda^{\mu\nu}M_{\mu\nu})\in\mathrm{O}(2,3)} g_0 g(x)=g(x')\exp[\tfrac{1}{2}\mathrm{i}u^{\mu\nu}(g_0,x)M_{\mu\nu}]. \tag{2.5}$$

Shifts with $g_0\in\mathrm{O}(1,3)$ induce on x_{μ} usual Lorentz transformations which form a little (stability or structure) subgroup of realisation (2.5). Shifts with $g_0=\exp(\mathrm{i}\lambda^{\mu}R_{\mu})$ result in nonlinear transformations

$$\delta_R x_{\mu}=\tfrac{1}{2}[\lambda_{\mu}+2m^2(\lambda x)x_{\mu}-m^2x^2\lambda_{\mu}]. \tag{2.6}$$

Transformation properties of Lorentz irreducible fields $\phi_k(x)$ with respect to realisation (2.5) can be defined naturally following the induced representation method

$$\phi'_k(x')=[\exp(\tfrac{1}{2}\mathrm{i}u^{\mu\nu}(g_0,x)J_{\mu\nu})]_{kl}\phi_l(x) \tag{2.7}$$

where $(J_{\mu\nu})_{kl}$ are matrices representing generators of the little group O(1,3) on fields $\phi_k(x)$. For the infinitesimal O(2,3) translations (2.6) the functions $u^{\mu\nu}(\lambda, x)$ are given by

$$u^{\mu\nu}(\lambda,x)=m^2(\lambda^{\mu}x^{\nu}-\lambda^{\nu}x^{\mu})=\tfrac{1}{2}(\partial^{\mu}\delta_R x^{\nu}-\partial^{\nu}\delta_R x^{\mu}). \tag{2.8}$$

At this point, one essential remark should be made. In so far as the subgroup O(1,3) of O(2,3) is identified with the physical Lorentz group the law (2.7) is the most general

† A similar parametrisation of cosets O(2,3)/O(1,3) has also been used by Gürsey and Marchildon (1978a, b). However, their treatment of O(2,3) (and OSp(1,4)) differs essentially from ours. Together with MacDowell and Mansouri (1977) and Chamseddine (1977, 1978) they regard these groups as purely gauge, i.e. as acting in some internal tangent space. In such a treatment parameters of cosets are fields over usual Minkowski space–time, which is not affected by group transformations at all.

‡ These coordinates also determine a particular parametrisation of the coset space O(2,3)/O(1,3). Their explicit relation to z_{μ} was given by Keck (1975) and Zumino (1977).

O(2,3) transformation law for fields defined over anti-de Sitter space (up to a change of coordinates). For instance, given some linear O(2,3) multiplet $\Phi(x)$ such that $\Phi(x) \xrightarrow{O(2,3)} \Phi'(x') = \bar{g}\Phi(x)$ where $\bar{g}$ is an appropriate matrix representation of O(2,3), it can be decomposed into the direct sum of Lorentz irreducible fields with transformation properties (2.7) by means of the equivalence replacement $\Phi(x) \to \tilde{\Phi}(x) = \exp(-iz^{\mu}(x)\bar{R}_{\mu})\Phi(x)$ where $\bar{R}_{\mu}$ are matrices of generators R_{μ} in the representation $\bar{g}$. This phenomenon is a particular manifestation of the relationship between linear and nonlinear group realisations (general theorems are given by Coleman *et al* 1969). As an important example, we write down explicitly the equivalency transformation by which some O(2,3) spinor $\psi^{s}_{\alpha}(x)$ $(\bar{R}_{\mu} = \frac{1}{2}m\gamma_{\mu})$ is expressed in terms of components $\psi_{\alpha}(x)$ comprising a Lorentz spinor

$$\psi^{s}_{\alpha}(x) = [\exp(\tfrac{1}{2}imz^{\mu}(x)\gamma_{\mu})]^{\beta}_{\alpha}\psi_{\beta}(x) = [\tfrac{1}{2}a(x)]^{1/2}(1 + imx^{\mu}\gamma_{\mu})^{\beta}_{\alpha}\psi_{\beta}(x) \equiv \Lambda^{\beta}_{\alpha}(x)\psi_{\beta}(x) \quad (2.9)$$

with

$$a(x) = 2/(1 + m^2x^2). \quad (2.10)$$

The transformation (2.9) has been used earlier by Dirac (1935) and Zumino (1977), without, however, explanation of its group meaning.

Note also that we might choose O(2,3) transformations to coincide with those of the O(2,3) subgroup of the usual conformal group (with identification $R_{\mu} = \frac{1}{2}(P_{\mu} - m^2K_{\mu})$). Again, an equivalence mapping which brings them into the standard form (2.7) exists (see appendix 3).

Let us define now the covariant differentials and derivatives of fields $\phi_k(x)$. This can be done quite simply using the method of Cartan differential forms. In our case, these forms are found from the decomposition

$$\begin{aligned} g^{-1}(x)\,dg(x) &= i\mu^{\nu}_{s}(x, dx)R_{\nu} + \tfrac{1}{2}i\nu^{\mu\rho}_{s}(x, dx)M_{\mu\rho} \\ &\equiv ia(x)\,dx^{\nu}R_{\nu} - im^2a(x)x^{\mu}\,dx^{\rho}M_{\mu\rho}. \end{aligned} \quad (2.11)$$

The form $\mu^{\nu}_{s} = a(x)\,dx^{\nu}$ transforms homogeneously under shifts (2.5) as the Lorentz four-vector, with parameters $u^{\mu\nu}(g_0, x)$, and is thus the covariant differential of the coordinate x^{ν}. The inhomogeneously transforming form $\nu^{\mu\rho}_{s} = -m^2a(x)(x^{\mu}\,dx^{\rho} - x^{\rho}\,dx^{\mu})$ (the Lorentz connection) enters into the covariant differentials of fields $\phi_k(x)$:

$$\nabla\phi_k(x) = d\phi_k(x) + \tfrac{1}{2}i\nu^{\mu\rho}_{s}(x, dx)(J_{\mu\rho})_{kl}\phi_l(x) \quad (2.12)$$

which transform as $\phi_k(x)$ themselves. The covariant derivatives $\nabla_{\rho}\phi_k(x)$ are naturally defined as coefficients of the expansion of $\nabla\phi_k(x)$ in forms μ^{ρ}_{s}:

$$\nabla\phi_k(x) = \mu^{\rho}_{s}(x, dx)\nabla_{\rho}\phi_k(x), \quad (2.13)$$

whence

$$\nabla_{\rho}\phi_k(x) = a^{-1}(x)\partial_{\rho}\phi_k(x) - im^2x^{\mu}(J_{\mu\rho})_{kl}\phi_l(x). \quad (2.14)$$

It is worth noting the useful formula†

$$[\nabla_{\rho}, \nabla_{\lambda}] = -im^2J_{\rho\lambda}, \quad (2.15)$$

† Hereafter we mean that the matrix part of each covariant derivative in operators of the type $\nabla_{\rho}\nabla_{\mu}\ldots\nabla_{\lambda}$ acts on all free Lorentz indices to the right of it, including vector indices of covariant derivatives.

which can be deduced either directly or using the following general method. One should evaluate the commutator of two independent covariant differentials (2.12), extract from both sides of the obtained identity independent products of forms μ_s^ρ, taking into account the Maurer–Cartan structure equations for μ_s^ρ, $\nu_s^{\rho\mu}$, and, finally, identify coefficients of these products (the structure equations for anti-de Sitter space are readily extracted from more general ones for the superspace OSp(1,4)/O(1,3) which are given in § 2.3). Relations (2.15) are formally similar to the commutators between generators R_μ. The essential difference is that only matrix, vierbein parts of Lorentz generators occur in the right-hand side of (2.15), in contrast to the whole $M_{\mu\nu}$ in (2.1a). This fact can easily be understood from the following consideration: as the LHS of (2.15) is manifestly O(2,3)-covariant the RHS must be of the same type ($J_{\mu\nu}\phi_k(x)$ is again a O(2,3)-covariant field while $M_{\mu\nu}\phi_k(x)$ is not).

Obtained formulae allow us to construct all objects relevant to the geometry of anti-de Sitter space. The contraction of two forms μ_s^ν gives the invariant interval

$$ds^2 = \mu_s^\mu \mu_{s\mu} = g^s_{\mu\nu}(x)\,dx^\mu\,dx^\nu = a^2(x)\eta_{\mu\nu}\,dx^\mu\,dx^\nu, \tag{2.16}$$

and the outer product of four forms μ_s^ν the invariant volume element

$$\mathscr{D}M_s = \mu_s^{\nu_1}\wedge\mu_s^{\nu_2}\wedge\mu_s^{\nu_3}\wedge\mu_s^{\nu_4} = d^4x(-\|g^s_{\mu\nu}\|)^{1/2} = d^4x\,a^4(x). \tag{2.17}$$

One immediately observes that $g^s_{\mu\nu}(x) = a^2(x)\eta_{\mu\nu}$ plays the role of anti-de Sitter metric, $a(x)\eta_{\mu\nu}$ being an appropriate vierbein. The curvature tensor can be defined now in the standard fashion. As a matter of fact, its components are given already by equation (2.15):

$$R^{\rho\lambda}_{\mu\nu} = -m^2(\delta^\rho_\mu\delta^\lambda_\nu - \delta^\lambda_\mu\delta^\rho_\nu),$$

taking into account that (2.14) and (2.15) are particular cases of well-known general covariance formulae (corresponding to the above special choice of vierbein).

In this connection, it is instructive to look at the group structure given by equations (2.6) and (2.7) from the general relativity standpoint. As is known, the invariance group of Einstein's theory in the vierbein interpretation is the semi-direct product of two infinite-parameter groups: the general covariance group (acting only on x_μ and world tensor indices) and the gauge (vierbein) Lorentz group which acts in a tangent space and does not affect the space–time coordinates. Both of them include O(1,3) subgroups, the physical Lorentz group being identified with the diagonal in the direct product of these O(1,3). By construction it does not distinguish world and vierbein indices. Clearly, transformations (2.6) together with Lorentz rotations of x_μ form a subgroup O(2,3) in the general covariance group. At the same time, it is not hard to see that the vierbein group itself does not contain a O(2,3)-subgroup. But one is still able to find x-dependent vierbein transformations which, being combined with shifts (2.6), close to form O(2,3) including as O(1,3)-subgroup the physical Lorentz group. The realisation thus obtained is just that one given by the law (2.7). In other words, the realisation of O(2,3) constructed here by the general procedure can also be regained by identifying certain parameters of the general covariance group with those from the gauge vierbein Lorentz group.

In such an interpretation, Lorentz indices of fields with the standard transformation law (2.7) should be regarded as coming from the tangent space because the $\phi_k(x)$ undergo in them only Lorentz rotations (x-dependent in the case of O(2,3)/O(1,3) transformations). It is interesting that for fields with tensor indices equivalent forms of the O(2,3)-transformation are also possible which coincide in appearance with transformation rules of world tensors in general relativity. More specifically, attaching to some vector index the factor $a(x)$ (vierbein) or $a^{-1}(x)$ (inverse vierbein) one obtains a field which transforms with respect to this index as the contra- or covariant world vector (with $\delta_R x_\mu$ from equation (2.6)). The equivalence field redefinition of this kind is a particular case of the generalised Weyl transformation (A3.1) corresponding to certain values of the weight number n therein.

To conclude, having expressions for the covariant derivatives of the fields $\phi_k(x)$ and for the O(2,3)-invariant volume element (which is the measure of integration over the space O(2,3)/O(1,3)) we can construct O(2,3)-invariant Lagrangian densities of any desirable structure in $\phi_k(x)$. Also, the problem of reduction of $\phi_k(x)$ to O(2,3)-irreducible pieces can be solved (by representing two Casimir operators of O(2,3) in terms of covariant derivatives (2.14) and proceeding further like Grensing (1977) in his analysis of O(1,4)-de Sitter fields).

As $m \to 0$, all the expressions obtained reduce to their trivial Minkowski analogues (within our definition of anti-de Sitter coordinate, the complete correspondence with Minkowski space arises upon rescaling $x_\mu \to \frac{1}{2}x_\mu$).

2.3. *Extension to the superspace OSp(1,4)/O(1,3)*

To develop the covariant superfield technique for OSp(1,4)-symmetry, we shall follow, as before, general recipes of the theory of group realisations in coset spaces and represent OSp(1,4) by left shifts in the superspace OSp(1,4)/O(1,3). For cosets of OSp(1,4) over the group O(1,3) we take the parametrisation

$$G(x, \theta) = \mathrm{O}(2,3)/\mathrm{O}(1,3) \,.\, \mathrm{OSp}(1,4)/\mathrm{O}(2,3) = g(x) \exp[\mathrm{i}(1 - \tfrac{1}{3}m\bar{\theta}\theta)\bar{\theta}Q] \tag{2.18}$$

where θ_α are the Grassmann coordinates associated with the generator Q_α and comprising a Majorana spinor. The parametrisation (2.18) differs from that adopted by Keck (1975) and Zumino (1977), besides the different choice of the coordinate system in the space O(2,3)/O(1,3), by the opposite arrangement of even and odd factors. We adhere to this sequence in order that, under the left shifts belonging to the subgroup O(2,3), the coordinates θ_α transform according to the induced representation law (2.7), i.e. like a Lorentz spinor. With this choice, different θ-monomials do not mix under O(2,3)-transformations, and as a consequence, components of OSp(1,4) superfields have the uniform transformation properties (2.7) in all their Lorentz indices. At the same time, making use of the parametrisation by Keck (1975) and Zumino (1977), the spinor coordinate (denoted here by θ^{s}) behaves like an O(2,3)-spinor, i.e. it transforms under O(2,3)-translations as $\delta\theta^{\mathrm{s}} = \frac{1}{2}\mathrm{i}m\lambda^\rho\gamma_\rho\theta^{\mathrm{s}}$. For this reason, components of corresponding superfields, in indices associated with θ-monomials, form linear multiplets of O(2,3), which seems to us less convenient because of the lack both of uniformity and explicit correspondence with the ordinary supersymmetry. The connection of our coordinates x_μ, θ_α with those of Keck (1975) and Zumino (1977) is given by equation (2.4) and by the formula of the type (2.9),

$$\theta^{\mathrm{s}}_\alpha = \Lambda^\beta_\alpha(x)\theta_\beta(1 - \tfrac{1}{3}m\bar{\theta}\theta). \tag{2.19}$$

An additional canonical redefinition of the Grassmann coordinate with the help of the θ-dependent factor $(1 - \frac{1}{3}m\bar{\theta}\theta)$ in (2.18) is made to simplify subsequent formulae (it does not alter the O(2,3)-properties of θ_α).

Transformation properties of the superspace OSp(1,4)/O(1,3) and superfields $\Phi_k(x, \theta)$ defined over it (k is the Lorentz index) with respect to an arbitrary OSp(1,4)-transformation are specified by the formulae (compare with (2.5) and (2.7))

$$G(x, \theta) \rightarrow G_0 G(x, \theta) = G(x', \theta') \exp(\tfrac{1}{2}\mathrm{i} W^{\mu\nu}(G_0, x, \theta) M_{\mu\nu}), \tag{2.20}$$

$$\Phi_k(x, \theta) \rightarrow \Phi'_k(x', \theta') = [\exp(\tfrac{1}{2}\mathrm{i} W^{\mu\nu}(G_0, x, \theta) J_{\mu\nu})]_{kl} \Phi_l(x, \theta), \tag{2.21}$$

with $(J_{\mu\nu})_{kl}$ again being the matrix realisation of generators $M_{\mu\nu}$.

Clearly, for $G_0 = g_0 \in \mathrm{O}(2,3)$ transformations (2.20) and (2.21) reduce to (2.5) and (2.7). In particular, $W^{\mu\nu}(g_0, x, \theta) = u^{\mu\nu}(g_0, x)$, and generators $M_{\mu\nu}$ and R_λ in the realisation on superfields $\Phi_k(x, \theta)$ are simply

$$\begin{aligned} M_{\mu\nu} &= \mathrm{i}(x_\mu \partial_\nu - x_\nu \partial_\mu) + \tfrac{1}{2}\bar{\theta}\sigma_{\mu\nu}\, \partial/\partial\bar{\theta} + J_{\mu\nu}, \\ R_\lambda &= \mathrm{i}[\tfrac{1}{2}(1 - m^2 x^2)\delta_\lambda^\nu + m^2 x_\lambda x^\nu]\partial_\nu + \tfrac{1}{2} m^2 x^\nu \bar{\theta}\sigma_{\lambda\nu}\, \partial/\partial\bar{\theta} - m^2 x^\nu J_{\nu\lambda}. \end{aligned} \tag{2.22}$$

Much more involved is the structure of the odd OSp(1,4)-transformations generated by shifts with $G = \mathrm{e}^{\mathrm{i}\bar{\varepsilon}Q}$. Making use of the commutation relations (2.1) and formulae of Zumino (1977), through rather cumbersome calculations we have found that the generators Q_α realised on superfields $\Phi_k(x, \theta)$ are of the form

$$\begin{aligned} Q_\alpha = (1 - \tfrac{1}{4}m\bar{\theta}\theta)\Lambda_\alpha^\beta(x)[\![\mathrm{i}\{(1 + \tfrac{1}{4}m\bar{\theta}\theta)\delta_\beta^\gamma + m\theta_\beta\bar{\theta}^\gamma - \tfrac{1}{4}m(\gamma^\nu\theta)_\beta[\bar{\theta}(\gamma_\nu - mx^\rho\sigma_{\rho\nu})]^\gamma\}\, \partial/\partial\bar{\theta}^\gamma \\ - \tfrac{1}{2}a^{-1}(x)(\gamma^\nu\theta)_\beta\partial_\nu + \tfrac{1}{4}\mathrm{i}m[(2mx_\nu\gamma_\mu + \sigma_{\nu\mu})\theta]_\beta J^{\nu\mu}]\!] \end{aligned} \tag{2.23}$$

where the matrix $\Lambda_\alpha^\beta(x)$ and function $a(x)$ are introduced by equations (2.9) and (2.10). The differential parts of generators (2.22) and (2.23) transform arguments x_μ and θ_α, while the matrix ones change the superfield form via the Lorentz rotation with x- and θ-dependent parameters. In the contraction limit, (2.22) and (2.23) convert into the generators of usual supersymmetry (with additional rescaling $x_\mu \rightarrow \tfrac{1}{2}x_\mu$).

We now turn to the construction of covariants of the realisation given by transformation laws (2.20) and (2.21). The relevant Cartan forms are introduced by the relation which is a direct extension of the decomposition (2.11):

$$G^{-1}(x, \theta)\, \mathrm{d}G\, (x, \theta) = \mathrm{i}\bar{\tau}(x, \theta, \mathrm{d}x, \mathrm{d}\theta)Q + \mathrm{i}\mu^\nu(x, \theta, \mathrm{d}x, \mathrm{d}\theta)R_\nu + \tfrac{1}{2}\mathrm{i}\nu^{\rho\lambda}(x, \theta, \mathrm{d}x, \mathrm{d}\theta)M_{\rho\lambda}. \tag{2.24}$$

The forms μ^ν, $\bar{\tau}^\alpha$ are easily verified to transform under shifts (2.20) according to the general law (2.21), independently of each other. Thus they have the meaning of covariant differentials of the coordinates x_ν and θ_α. The inhomogeneously transforming form $\nu^{\rho\lambda}$ is nothing but the connection over the Lorentz group (which is the structure group of the present realisation). This form determines the covariant differentials of the superfields:

$$\mathscr{D}\Phi_k(x, \theta) = \mathrm{d}\Phi_k(x, \theta) + \tfrac{1}{2}\mathrm{i}\nu^{\rho\lambda}(J_{\rho\lambda})_{kl}\Phi_l(x, \theta). \tag{2.25}$$

Inhomogeneity of its transformation is just such as to compensate the non-covariant term which arises from $\mathrm{d}\Phi_k(x, \theta)$ when $\Phi_k(x, \theta)$ undergoes the transformation (2.21). As a result, $\mathscr{D}\Phi_k(x, \theta)$ transforms like $\Phi_k(x, \theta)$ itself.

Explicitly, the forms $\bar{\tau}^\beta$, μ^ν, $\nu^{\rho\lambda}$ are as follows:

$$\begin{aligned} \bar{\tau}^\beta &= (1 - \tfrac{1}{2}m\bar{\theta}\theta)[\![\mathrm{d}\bar{\theta}^\alpha\{[1 + \tfrac{1}{32}gm^2(\bar{\theta}\theta)^2]\delta_\alpha^\beta - \tfrac{1}{2}m\theta_\alpha\bar{\theta}^\beta\} - \tfrac{1}{2}\mathrm{i}ma(x)\, \mathrm{d}x_\mu[\bar{\theta}(\gamma^\mu - mx_\nu\sigma^{\nu\mu})]^\beta]\!] \\ \mu^\nu &= a(x)\, \mathrm{d}x^\nu + \tfrac{1}{2}\mathrm{i}(1 - \tfrac{1}{4}m\bar{\theta}\theta)\bar{\tau}\gamma^\nu\theta, \\ \nu^{\rho\lambda} &= \tfrac{1}{2}\mathrm{i}m(1 - \tfrac{1}{4}m\bar{\theta}\theta)\bar{\tau}[\sigma^{\rho\lambda} + m(x^\rho\gamma^\lambda - x^\lambda\gamma^\rho)]\theta - m^2(x^\rho\mu^\lambda - x^\lambda\mu^\rho). \end{aligned} \tag{2.26}$$

In constructing the covariant derivatives of superfields we shall follow closely the pure anti-de Sitter case, namely, we extract from $\mathscr{D}\Phi_k(x, \theta)$ (2.25) the covariant differentials of the coordinates x_ν, θ_α, i.e. the forms μ^ν, $\bar{\tau}^\alpha$, and identify with the covariant derivatives, respectively vector and spinor, the coefficients of these forms:

$$\mathscr{D}\Phi_k(x, \theta) = \mu^\nu \hat{\nabla}_\nu \Phi_k(x, \theta) + \bar{\tau}^\alpha \hat{\mathscr{D}}_\alpha \Phi_k(x, \theta). \tag{2.27}$$

The objects thus defined are manifestly OSp(1,4)-covariant by construction. With the explicit expressions for Cartan forms (2.26), definition (2.27) implies

$$\hat{\nabla}_\nu = a^{-1}(x)\partial_\nu + \tfrac{1}{2}\mathrm{i}m\bar{\theta}(\gamma_\nu - mx^\mu\sigma_{\mu\nu})\,\partial/\partial\bar{\theta} - \mathrm{i}m^2 x^\mu J_{\mu\nu}, \tag{2.28}$$

$$\hat{\mathscr{D}}_\alpha = (1 - \tfrac{1}{4}m\bar{\theta}\theta)\{[(1+\tfrac{3}{4}m\bar{\theta}\theta)\delta_\alpha^\beta + \tfrac{1}{2}m\theta_\alpha\bar{\theta}^\beta]\,\partial/\partial\bar{\theta}^\beta - \tfrac{1}{2}\mathrm{i}(\gamma^\mu\theta)_\alpha\hat{\nabla}_\mu - \tfrac{1}{4}m(\sigma^{\mu\nu}\theta)_\alpha J_{\mu\nu}\}. \tag{2.29}$$

As $m \to 0$, these operators contract into the usual vector and spinor covariant derivatives of 'flat' supersymmetry. Let us point out that $\mathrm{d}x_\mu$ and ∂_μ enter into the forms (2.26) and covariant derivatives (2.28) and (2.29) only through their O(2,3)-counterparts μ_s^μ and ∇_μ, thus ensuring correct O(2,3)-properties for (2.26), (2.28) and (2.29). Note also that the pieces of the forms (2.26) independent of x_μ and $\mathrm{d}x_\mu$ coincide with the Cartan forms for the realisation of OSp(1,4) in the purely Grassmannian coset space OSp(1,4)/O(2,3).

Though the structure of covariant derivatives (2.28) and (2.29) is rather complicated their commutator algebra turns out, beyond expectation, almost as simple as in the case of usual supersymmetry:

$$\begin{aligned}
&[\hat{\nabla}_\mu, \hat{\nabla}_\nu] = -\mathrm{i}m^2 J_{\mu\nu},\\
&\{\hat{\mathscr{D}}_\alpha, \hat{\mathscr{D}}_\beta\} = -\tfrac{1}{2}m(\sigma^{\mu\nu}C)_{\alpha\beta}J_{\mu\nu} + (1/\mathrm{i})(\gamma^\nu C)_{\alpha\beta}\hat{\nabla}_\nu,\\
&[\hat{\nabla}_\mu, \hat{\mathscr{D}}_\alpha] = (m/2\mathrm{i})(\gamma_\mu\hat{\mathscr{D}})_\alpha,
\end{aligned} \tag{2.30}$$

multiplication of covariant derivatives being understood here in the sense explained in the footnote to formula (2.15). To learn what these (anti) commutators are, we have taken advantage of the general method mentioned after equation (2.15) (the straightforward computation of them is also possible, but it involves a lot of tedious labour). In application to the present case, that method consists of evaluating an antisymmetrised second-order covariant differential of $\Phi_k(x, \theta)$ and equating afterwards coefficients of independent products of the forms μ^ν, $\bar{\tau}^\alpha$ on both sides of the resulting identity. This procedure essentially exploits the Maurer–Cartan structure equations for the superspace OSp(1,4)/O(1,3):

$$\begin{aligned}
&\mathscr{D}_2\bar{\tau}(\mathrm{d}_1) - \mathscr{D}_1\bar{\tau}(\mathrm{d}_2) + (m/2\mathrm{i})[\bar{\tau}(\mathrm{d}_1)\gamma^\nu\mu_\nu(\mathrm{d}_2) - \bar{\tau}(\mathrm{d}_2)\gamma^\nu\mu_\nu(\mathrm{d}_1)] = 0,\\
&\mathscr{D}_2\mu_\nu(\mathrm{d}_1) - \mathscr{D}_1\mu_\nu(\mathrm{d}_2) - \mathrm{i}\bar{\tau}(\mathrm{d}_1)\gamma_\nu\tau(\mathrm{d}_2) = 0,\\
&\mathrm{d}_2\nu^{\rho\lambda}(\mathrm{d}_1) - \mathrm{d}_1\nu^{\rho\lambda}(\mathrm{d}_2) - [\nu_\gamma^\rho(\mathrm{d}_2)\nu^{\gamma\lambda}(\mathrm{d}_1) - \nu_\gamma^\rho(\mathrm{d}_1)\nu^{\gamma\lambda}(\mathrm{d}_2)]\\
&\qquad - m^2[\mu^\rho(\mathrm{d}_1)\mu^\lambda(\mathrm{d}_2) - \mu^\rho(\mathrm{d}_2)\mu^\lambda(\mathrm{d}_1)] - \mathrm{i}m\bar{\tau}(\mathrm{d}_1)\sigma^{\rho\lambda}\tau(\mathrm{d}_2) = 0,
\end{aligned} \tag{2.31}$$

which are derived in appendix 4.

It follows from (2.30) that, in contrast to the case of ordinary ('flat') supersymmetry, OSp(1,4)-covariant derivatives $\hat{\nabla}_\mu$, $\hat{\mathscr{D}}_\alpha$ do not generate the algebra isomorphic to the initial one (2.1). Indeed, relations (2.30), like their O(2,3)-counterparts (2.15) contain only tangent-space pieces of Lorentz generators. Besides, relative signs between different structures on the right-hand side of (2.30) are somewhat distinguished from those appearing in (2.1). Strictly speaking, relations (2.30) (as well as (2.15)) should not be referred to as introducing any algebra in its conventional meaning because (anti) commutators between covariant derivatives are defined in (2.30) quite differently from those between infinitesimal group generators† (see the footnote to equation (2.15)). Rather, equations (2.30) have to be regarded as an equivalent form of the structure equations (2.31). If, nevertheless, one attempts to treat (2.28) and (2.29) as generators of certain superfield transformations and begins to commute (anticommute) them in the usual way, one immediately observes that they do not form any closed superalgebra (even together with OSp(1,4)-generators (2.22), (2.23)) and it is not clear to which more extensive (finite-dimensional) superalgebra they could pertain. Note that (anti) commutators (in the usual sense) of $\hat{\nabla}_\mu$, $\hat{\mathscr{D}}_\alpha$ with OSp(1,4)-generators are again the same objects, but Lorentz rotated according to the general rule (2.21). For instance,

$$\{\hat{\mathscr{D}}_\alpha, Q_\beta\} = \frac{1}{4}\left[\frac{\partial}{\partial\bar{\epsilon}^\beta} W^{\mu\nu}(\epsilon, x, \theta)\right]_{\epsilon=0} (\sigma_{\mu\nu})_\alpha^\gamma \hat{\mathscr{D}}_\gamma \qquad (=0 \text{ as } m \to 0).$$

In this way, OSp(1,4)-covariance of $\hat{\nabla}_\mu$, $\hat{\mathscr{D}}_\alpha$ displays itself at the commutator level. Recall that the covariant spinor derivative of usual supersymmetry commutes with the 4-translation generator P_μ and anticommutes with the spinor generator. It can be defined through the Cartan forms method or, alternatively, as the generator of right supertranslations. In the OSp(1,4)-case, only the first approach appears constructive because the right action of OSp(1,4) on the cosets OSp(1,4)/O(1,3) does not commute with the left one (Keck 1975). Covariant derivatives (2.28) and (2.29) cannot be identified certainly with generators of the right OSp(1,4)-transformations since, as pointed out above, their 'algebra' is not closed with respect to the operation of usual (anti) commuting.

Now, let us dwell on the question of how to interpret the obtained formulae in the context of super-Riemannian geometry. We introduce the supervierbein $E(x, \theta)$, namely the supermatrix relating the Cartan 'superform' $w^N \equiv (\mu^\nu, \bar{\tau}^\alpha)$ to the 'superdifferential' $\mathrm{d}z^M \equiv (\mathrm{d}x^\nu, \mathrm{d}\bar{\theta}^\alpha)$:

$$w^N \equiv (\mu^\nu, \bar{\tau}^\alpha) = (\mathrm{d}x^\rho, \mathrm{d}\bar{\theta}^\beta)\begin{pmatrix} A_\rho^\nu(x, \theta) & B_\rho^\alpha(x, \theta) \\ C_\beta^\nu(x, \theta) & D_\beta^\alpha(x, \theta)\end{pmatrix} \equiv \mathrm{d}z^M E_M^N(z) \qquad (2.32)$$

and, respectively, the inverse supervierbein $E^{-1}(x, \theta)$; then we may rewrite the covariant derivatives $\hat{\nabla}_\rho$, $\hat{\mathscr{D}}_\alpha$ in the parametrisation-independent form in which they combine into the single object, OSp(1,4)-covariant superderivative $\hat{\nabla}_N = (E^{-1})_N^M \partial_M^+ \ldots$. Without giving it explicitly, we point out only that $\hat{\nabla}_N$ is a particular case of the generally covariant superderivative over the superspace with the Lorentz group as the structure group (for the general definition of such objects see e.g. Wess (1978)). This fact is quite natural because the superspace OSp(1,4)/O(1,3), as well as the standard one, is a special case of the superspace mentioned above. Correspondingly, the realisation (2.20), (2.21) can be embedded into the infinite-dimensional supergroup which is given on this general superspace and includes arbitrary reparametrisations of x_μ, θ_α together with purely gauge x,θ-dependent Lorentz rotations of external superfield indices. Thus the situation is strongly reminiscent of that taking place for anti-de Sitter space alone.

† In the usual supersymmetry, due to the absence of vierbein parts in the corresponding covariant derivatives, both definitions completely coincide. This is the reason why these derivatives form the true superalgebra (it is reproduced, of course, from (2.30) in the contraction limit).

In particular, Lorentz indices of all quantities which are subjected to the transformation rule (2.21) originate from the tangent space (the same is true, of course, for the generalised indices of the 'superform' $w^N \equiv (\mu^\nu, \bar{\tau}^\alpha)$ and the OSp(1,4)-covariant 'superderivative' $\hat{\nabla}_N \equiv (\hat{\nabla}_\rho, \hat{\mathscr{D}}_\alpha))$ while indices of coordinates $z^M = (x^\nu, \bar{\theta}^\alpha)$, their differentials $dz^M = (dx^\nu, d\bar{\theta}^\alpha)$ and related indices of direct and inverse supervierbeins E, E^{-1} behave under shifts (2.20) as the 'superworld' ones. Again, in close parallel with the O(2,3)-case, it should be realised that (2.21) is the most general OSp(1,4)-transformation law for superfields defined over the superspace OSp(1,4)/O(1,3) once the O(1,3)-subgroup of OSp(1,4) is supposed to be the physical Lorentz group. All other forms of the transformation rule reduce to (2.21) through the equivalence superfield redefinition of one kind or another. For instance, we may change OSp(1,4)-transformation properties of superfields $\Phi_k(x, \theta)$ by contraction of certain of their indices with supervierbeins E, E^{-1} (see also appendix 3).

Disposing of the explicit expressions (2.26) for the Cartan forms, we may readily construct all geometrical characteristics of the superspace OSp(1,4)/O(1,3). As in the case of conventional superspace, there exist three independent invariant 'intervals': $\mu^\nu\mu_\nu$, $\bar{\tau}\tau$, $\bar{\tau}\gamma_5\tau$. The components of the curvature and torsion supertensors are extracted most easily from the relations (2.30): they are given, respectively, by coefficients of the matrix $\frac{1}{2}iJ_{\mu\nu}$ and covariant derivatives on the RHS of (2.30). In obvious notation

$$R^{\lambda\rho}_{\mu\nu} = -m^2(\delta^\lambda_\mu\delta^\rho_\nu - \delta^\rho_\mu\delta^\lambda_\nu), \qquad R^{\lambda\rho}_{\alpha\beta} = im(\sigma^{\lambda\rho}C)_{\alpha\beta}, \qquad T^\mu_{\alpha\beta} = -i(\gamma^\mu C)_{\alpha\beta},$$
$$T^\beta_{\mu\alpha} = -T^\beta_{\alpha\mu} = -\tfrac{1}{2}im(\gamma_\mu)^\beta_\alpha,$$

all other components being zero. Here, all indices refer to the tangent space (in the sense indicated above). The same results follow from the consideration of the structure equations (2.31). As is expected, in the limit $m \to 0$ there survives only the supertorsion component $T^\nu_{\alpha\beta}$ associated with the superspace of ordinary supersymmetry.

To conclude this section, we calculate the OSp(1,4)-invariant measure of integration over the superspace OSp(1,4)/O(1,3) (the invariant volume element), the knowledge of which is important for constructing OSp(1,4)-invariant Lagrangians. It is defined in a standard manner (Arnowitt *et al* 1975) through a superdeterminant (Berezinian) of the supervierbein E:

$$\text{Ber}\, E = \det(A - BD^{-1}C)\det D^{-1}.$$

Substituting for matrices A, B, C, D their explicit expressions which are straightforwardly extracted from formulae (2.26), we find

$$\mathscr{D}M \equiv d^4x\, d^4\theta\, \text{Ber}\, E = d^4x\, d^4\theta\, a^4(x)[1 + \tfrac{3}{2}m\bar{\theta}\theta + \tfrac{3}{8}m^2(\bar{\theta}\theta)^2]. \qquad (2.33)$$

The invariance of the measure (2.33) with respect to the transformations of coordinates x_μ and θ_α generated by (2.22) and (2.23) can be verified directly, by using the rules for changing variables in the Grassmann integrals (Pakhomov 1974). Its factorisation has the clear meaning: the factor $d^4x\, a^4(x)$ is the O(2,3)-invariant measure of integration over the space O(2,3)/O(1,3), and the remaining part coincides with the integration measure for the superspace OSp(1,4)/O(2,3) invariant with respect to the left action of OSp(1,4) in that superspace. Note that the full invariant volume of the space OSp(1,4)/O(2,3) obtained by integration over the measure $d^4\theta[1 + \frac{3}{2}m\bar{\theta}\theta + \frac{3}{8}m^2(\bar{\theta}\theta)^2]$ is $3m^2 > 0$, which is to be compared with the case of the coset space of the standard supergroup over the Poincaré group the volume of which is zero.

3. Splitting bases in superspace OSp(1,4)/O(1,3) and chiral representations of OSp(1,4)

In this section, we pass to the non-symmetric, splitting parametrisation of the superspace OSp(1,4)/O(1,3). The basic property of this parametrisation is that the right- and left-handed components of the corresponding Grassmann coordinate enter into superfield transformation rules in essentially different ways. Just as in the usual supersymmetry, the splitting basis appears in many cases more convenient and advantageous than the symmetric one we dealt with before.

3.1. Reduction of OSp(1,4) superfields

We begin with the problem of the reduction of a general OSp(1,4)-superfield

$$\Phi_k(x,\theta)=A_k(x)+\bar\theta\psi_k(x)+\tfrac14\bar\theta\theta F_k(x)+\tfrac14\bar\theta\gamma_5\theta G_k(x)$$
$$+\tfrac14\bar\theta\mathrm{i}\gamma_\mu\gamma_5\theta A_k^\mu(x)+\tfrac14\bar\theta\theta\,\bar\theta\chi_k(x)+\tfrac1{32}(\bar\theta\theta)^2D_k(x) \qquad (3.1)$$

subject to the standard OSp(1,4)-supertranslation law

$$\delta\Phi_k(x,\theta)=\mathrm{i}(\bar\epsilon Q\Phi(x,\theta))_k, \qquad (3.2)$$

where ϵ is an anticommuting constant spinor parameter, and Q_α is given by (2.23).

In the usual supersymmetry, superfields of the type (3.1) are known to be locally reducible (Salam and Strathdee 1975). The reduction is effected by imposing covariant conditions of first and higher orders in covariant derivatives (Salam and Strathdee 1975, Sokatchev 1975). One may attempt to proceed in an analogous way in the OSp(1,4)-case. The simplest covariant conditions of first order in the derivatives are now

$$[\tfrac12(1\mp\mathrm{i}\gamma_5)]_\alpha^\beta(\hat{\mathscr{D}}_\beta\Phi(x,\theta))_k=0, \qquad (3.3_\pm)$$

which directly generalise the well-known constraints isolating chiral representations in the usual supersymmetry.

Equations (3.3) are solved most simply when rewritten in component form. Examining the system of differential equations thus obtained, we have found that, unlike the case of 'flat' supersymmetry, it possesses non-trivial solutions ($\Phi_k\neq0$) not for any superfields but only for those which are transformed by one of the following representations of the Lorentz group:

$$D^{(p,0)} \qquad \text{for condition } (3.3_+), \qquad (3.4_+)$$

$$D^{(0,q)} \qquad \text{for condition } (3.3_-), \qquad (3.4_-)$$

where $D^{(p,q)}$ are matrices of non-unitary finite-dimensional representations of the Lorentz group, p and q are positive integers and half-integers (see, e.g., Gürsey 1964). The meaning of these constraints will become clear later (§ 3.3).

In the superfields $\Phi_{\pm k}(x,\theta)$ of the classes $(3.4_\pm)$ the conditions $(3.3_\pm)$ pick out as independent components $A_{\pm k}$, $F_{\pm k}$ and $\psi_{\pm k}$ (the latter, in the suppressed spinor index associated with θ_α, is either left- or right-handed depending on the lower sign) and express the remaining components in terms of the independent ones:

$$G_{\pm k}=\pm\mathrm{i}F_{\pm k}, \qquad A_{\pm k}^\mu=\pm\mathrm{i}(\nabla^\mu A_\pm)_k,$$
$$\chi_{\pm k}=-\mathrm{i}(\gamma^\mu\nabla_\mu\psi_\pm)_k-2m\psi_{\pm k}, \qquad D_{\pm k}=-(\nabla^\mu\nabla_\mu A_\pm)_k-8mF_{\pm k}, \qquad (3.5_\pm)$$

where ∇_μ is the O(2,3)-covariant derivative defined by (2.14). Note the similarity of ($3.5_\pm$) to the corresponding relations of standard supersymmetry. The analogy becomes most striking after going back to the superfield notation. We have checked that the solutions ($3.5_\pm$) admit the compact superfield representation

$$\Phi_{\pm k}(x, \theta) = [\exp(\mp\tfrac{1}{4}\bar{\theta}\gamma^\rho\gamma_5\theta\hat{\nabla}_\rho)]_{kl}T_l^\pm(x, \theta_\pm), \tag{$3.6_\pm$}$$

$$T_k^\pm(x, \theta_\pm) = A_{\pm k}(x) + \bar{\theta}_\pm\psi_{\pm k}(x) + \tfrac{1}{2}\bar{\theta}_\pm\theta_\pm F_{\pm k}(x), \tag{$3.7_\pm$}$$

where $\hat{\nabla}_\rho$ is the OSp(1,4)-covariant vector derivative (2.28) and $\theta_\pm = \frac{1}{2}(1 \pm i\gamma_5)\theta$. Relations (3.6) are seen to appear as a direct 'covariantisation' ($\partial_\rho \to \hat{\nabla}_\rho$) of familiar formulae of 'flat' supersymmetry which describe the transition to the symmetric basis in corresponding 'truncated' chiral superfields. These formulae are just the contraction limit ($m = 0$) of (3.6). Without loss of generality, one may put $\Phi_{+k}(x, \theta) = \Phi^*_{-k}(x, \theta)$ and $T_k^+(x, \theta_+) = (T_k^-(x, \theta_-))^*$ where the symbol * means involution (complex conjugation plus reversion of the order of anticommuting factors)†.

Thus, we come to the conclusion that superfields from the restricted set ($3.4_\pm$) possess invariant chiral subspaces. Those superfields which are transformed by the direct sum of representations $D^{(p,0)} \oplus D^{(0,p)}$ (they can be submitted to the reality condition) contain invariant subspaces of both chiralities. The simplest example is a real scalar superfield $\Phi(x, \theta)$. It contains two irreducible conjugated scalar OSp(1,4)-multiplets involving, as suggested by the field content of ($3.7_\pm$), eight real independent components. This fact has been established earlier by Keck (1975) through a straightforward analysis of the transformation properties of superfield components.

Further reduction of superfields (3.1) can be effected by imposing supplementary conditions of higher order in covariant derivatives with the structure dictated by the structure of Casimir operators of the supergroup OSp(1,4). The corresponding procedure will repeat, in its main steps, that one employed in the usual case (Salam and Strathdee 1975, Sokatchev 1975). However, the knowledge of supplementary conditions and projection operators which single out higher representations of OSp(1,4) is, in our opinion, rather of academic interest. Based on analogy with the conventional supersymmetry, it should be expected that in OSp(1,4)-invariant theories of real interest (OSp(1,4)-symmetric Yang–Mills theory, OSp(1,4)-supergravity, etc.) a minimal set of relevant fields will be automatically picked out on account of additional local invariances (with the elimination of a subset of auxiliary fields afterwards through the equations of motion).

3.2. *Chiral realisations of OSp(1,4)*

From the existence of the representations ($3.6_\pm$) it follows that OSp(1,4) can be realised on the 'truncated' chiral superfields $T_k^\pm(x, \theta_\pm)$. To find these realisations, we reduce ($3.6_\pm$) in analogy with the case of usual supersymmetry to certain nonlinear shifts of superfield arguments. For this purpose, we first unlink the matrix and differential parts in the operator $\exp(\mp\frac{1}{4}\bar{\theta}\gamma^\rho\gamma_5\theta\hat{\nabla}_\rho)$. This can easily be done using the Baker–Hausdorf formula and the basic property of Grassmann coordinates $(\theta)^5 = 0$. We have

$$\exp(\mp\tfrac{1}{4}\bar{\theta}\gamma^\rho\gamma_5\theta\hat{\nabla}_\rho) = \exp(\pm\tfrac{1}{4}\mathrm{i}m^2\bar{\theta}\gamma^\rho\gamma_5\theta x^\nu J^\pm_{\nu\rho})$$
$$\times\exp\{\mp\tfrac{1}{4}[\bar{\theta}\gamma^\rho\gamma_5\theta a^{-1}(x)\partial_\rho + 2\mathrm{i}m\bar{\theta}\theta\,.\,\bar{\theta}\gamma_5(1+\tfrac{3}{4}\mathrm{i}mx\gamma)\,\partial/\partial\bar{\theta}]\} \tag{3.8}$$

† Leaving aside mathematical rigorousness, in what follows we shall occasionally use the term 'complex conjugation' instead of 'involution', using the former everywhere to mean just the second operation.

where $J^{\pm}_{\nu\rho}$ are matrices of generators of the Lorentz group in the representations $(3.4_{\pm})$. Applying further the general identity $e^{f(z)\partial_z}\phi(z) = \phi(e^{f(z)\partial_z}z)$, we rewrite $(3.6_{\pm})$ as

$$\tilde{\Phi}_{\pm k}(x, \theta) \equiv [\exp(\mp\tfrac{1}{4}\mathrm{i}m^2\bar{\theta}\gamma^\rho\gamma_5\theta x^\nu J^{\pm}_{\nu\rho})]_{kl}\Phi_{\pm l}(x, \theta) = \begin{cases} T^+_k(x^{\mathrm{L}}, \theta^{\mathrm{L}}) \\ T^-_k(x^{\mathrm{R}}, \theta^{\mathrm{R}}) \end{cases} \qquad (3.9_{\pm})$$

where

$$\begin{pmatrix} x^{\mathrm{L}}_\mu \\ \theta^{\mathrm{L}} \end{pmatrix} = \exp\{-\tfrac{1}{4}[\bar{\theta}\gamma^\rho\gamma_5\theta a^{-1}(x)\partial_\rho + 2\mathrm{i}m\bar{\theta}\theta.\bar{\theta}\gamma_5(1+\tfrac{3}{4}\mathrm{i}mx\gamma)\,\partial/\partial\bar{\theta}]\}\begin{pmatrix} x_\mu \\ \theta_+ \end{pmatrix}$$
$$= \begin{pmatrix} [1+(m^2/16a(x))(\bar{\theta}\theta)^2]x_\mu - \tfrac{1}{4}\bar{\theta}\gamma_\mu\gamma_5\theta a^{-1}(x) \\ \theta_+ - \tfrac{1}{2}m\bar{\theta}\theta(\theta_+ + \tfrac{3}{4}\mathrm{i}mx\gamma\theta_-) \end{pmatrix} \qquad (3.10)$$

and $x^{\mathrm{R}}_\mu = (x^{\mathrm{L}}_\mu)^*$, $\theta^{\mathrm{R}} = C\bar{\theta}^{\mathrm{LT}}$ result from (3.10) simply by the change $\theta_+ \leftrightarrow \theta_-$. With these relations, and making use of the fact that the realisation of OSp(1,4) on superfields $\Phi_{\pm k}(x, \theta)$ (and on canonically related superfields $\tilde{\Phi}_{\pm k}(x, \theta)$) is known, we are now in a position to deduce the OSp(1,4)-transformation rules of chiral superfields $T^{\pm}_k$. Transformation properties of their arguments x^{L}_μ, $\theta^{\mathrm{L}}_\alpha$ and x^{R}_μ, $\theta^{\mathrm{R}}_\alpha$ are uniquely determined by the properties of x_μ, θ_α owing to the explicit connection (3.10) (and the analogous one between x_μ, θ_α and x^{R}_μ, $\theta^{\mathrm{R}}_\alpha$). As expected, these pairs of variables form invariant spaces with respect to the action of OSp(1,4). Under the Lorentz rotations and O(2,3) translations they behave like coordinates x_μ and θ_α. Their odd transformations are given by

$$\delta_Q x^{\mathrm{L}}_\mu = \mathrm{i}a^{-1}(x^{\mathrm{L}})\bar{\epsilon}\Lambda(x^{\mathrm{L}})\gamma_\mu\theta^{\mathrm{L}},$$
$$\delta_Q\bar{\theta}^{\mathrm{L}} = \bar{\epsilon}\Lambda(x^{\mathrm{L}})[1-\tfrac{1}{2}m\bar{\theta}^{\mathrm{L}}\theta^{\mathrm{L}}(1-\tfrac{3}{2}\mathrm{i}mx^{\mathrm{L}}_\mu\gamma^\mu)]\tfrac{1}{2}(1+\mathrm{i}\gamma_5) \qquad (3.11)$$

(infinitesimal variations of x^{R}_μ, $\bar{\theta}^{\mathrm{R}}_\alpha$ are of the same form up to the replacements $\mathrm{L}\to\mathrm{R}$, $\tfrac{1}{2}(1+\mathrm{i}\gamma_5)\to\tfrac{1}{2}(1-\mathrm{i}\gamma_5)$). The matrix parts of the OSp(1,4)-transformations of T^+_k and T^-_k can also be shown to depend, respectively, either on x^{L}_μ, $\theta^{\mathrm{L}}_\alpha$ or on x^{R}_μ, $\theta^{\mathrm{R}}_\alpha$ (at this point, it is significant that the generators $J^{\pm}_{\mu\nu}$ of the representations $(3.4_{\pm})$ do contain projectors $\tfrac{1}{2}(1\pm\mathrm{i}\gamma_5)$. The concrete structure of OSp(1,4)-generators realised on superfields $T^{\pm}_k$ is readily extracted from the general expressions for OSp(1,4)-generators in shifted bases given in appendix 1.

We point out that the shifted bosonic coordinates x^{L}, x^{R} are complex by the definition (3.10), and this is quite natural in view of the essentially complex character of the realisation (3.11) and its conjugate. But once the transformation properties of the component fields in the decomposition of chiral superfields $T^+_k(x^{\mathrm{L}}, \theta^{\mathrm{L}})$, $T^-_k(x^{\mathrm{R}}, \theta^{\mathrm{R}})$ in the variables θ^{L} or θ^{R} are specified, it no longer makes any difference whether these fields are defined over complex or real manifolds: the group properties of their infinitesimal variations remain unchanged. In other words, when symmetrically parametrised chiral superfields $\Phi_{\pm k}(x, \theta)$ in $(3.6_{\pm})$ transform according to the uniform rule (3.2) the 'truncated' superfields $T^{\pm}_k(x, \theta_{\pm})$ $(3.7_{\pm})$ undergo transformations of the same form as $T^+_k(x^{\mathrm{L}}, \theta^{\mathrm{L}})$ and $T^-_k(x^{\mathrm{R}}, \theta^{\mathrm{R}})$ do but with x_μ in place of x^{L}_μ, x^{R}_μ and θ_+, θ_- in place of θ^{L}, θ^{R}. The complexity of δx_μ in these transformation laws should not give rise to any trouble because, since all field transformations are performed at fixed arguments, it does not contradict the reality of x_μ and reflects merely the unremovable complexity of the components of chiral superfields.

For illustration, we write down the transformations induced by the supershifts (3.11) for the components of scalar chiral superfields $T^{\pm}(x, \theta_{\pm})$:

$$\begin{aligned} \delta_Q A_\pm &= \bar{\beta}\psi_\pm, \\ \delta_Q \psi_\pm &= \tfrac{1}{2}(1 \pm \mathrm{i}\gamma_5)(-\mathrm{i}\gamma^\mu \nabla_\mu A_\pm + F_\pm)\beta, \\ \delta_Q F_\pm &= \bar{\beta}(-\mathrm{i}\gamma^\mu \nabla_\mu \psi_\pm - m\psi_\pm), \end{aligned} \tag{3.12}$$

where $\beta = \Lambda^{-1}(x)\epsilon$. The laws (3.12) are the same as those we have found earlier (Ivanov and Sorin 1979b). In the contraction limit they become the usual transformation laws of scalar multiplets of 'flat' supersymmetry.

3.3. OSp(1,4)-superfields in chiral bases

To shed more light on the above results and, especially, to clarify the meaning of the restrictions ($3.4_\pm$) (which seem, at first sight, quite mysterious) we adopt here a more general point of view on the relation between the real superspace and chiral ones. Namely, we shall show that the change of variable (3.10) and its right-handed analogue naturally emerge within the coordinate transformations to the following new complex parametrisations of the cosets OSp(1,4)/O(1,3):

$$G(x, \theta) \to G^+(x^{\mathrm{L}}, \theta^{\mathrm{L}}, \eta^{\mathrm{R}}) = \mathrm{OSp}(1,4)/\mathrm{S}_- \,.\, \mathrm{S}_-/\mathrm{O}(1,3) = g(x^{\mathrm{L}})\, \mathrm{e}^{\mathrm{i}\bar{\theta}^{\mathrm{L}} Q_+}\, \mathrm{e}^{\mathrm{i}\bar{\eta}^{\mathrm{R}} Q_-} \tag{3.13_+}$$

and

$$G(x, \theta) \to G^-(x^{\mathrm{R}}, \theta^{\mathrm{R}}, \eta^{\mathrm{L}}) = \mathrm{OSp}(1,4)/\mathrm{S}_+ \,.\, \mathrm{S}_+/\mathrm{O}(1,3) = g(x^{\mathrm{R}})\, \mathrm{e}^{\mathrm{i}\bar{\theta}^{\mathrm{R}} Q_-}\, \mathrm{e}^{\mathrm{i}\bar{\eta}^{\mathrm{L}} Q_+}, \tag{3.13_-}$$

where $\mathrm{S}_\mp \propto (M_{\mu\nu}, Q_\mp)$ are the right- and left-handed Lorentz supergroups defined by the structure relations (2.2). Cumbersome calculations utilising the Baker–Hausdorf formula, structure relations (2.1)–(2.3), and the Grassmann nature of the coordinates θ_α indicate that $G(x, \theta)$ admits a representation in the two equivalent forms

$$G(x, \theta) = \begin{cases} G^+[x^{\mathrm{L}}(x, \theta), \theta^{\mathrm{L}}(x, \theta), \eta^{\mathrm{R}}(x, \theta)] \exp(-\tfrac{1}{4}\mathrm{i}m^2 \bar{\theta}\gamma^\rho\gamma_5\theta x^\nu M_{\nu\rho}) \\ G^-[x^{\mathrm{R}}(x, \theta), \theta^{\mathrm{R}}(x, \theta), \eta^{\mathrm{L}}(x, \theta)] \exp(\tfrac{1}{4}\mathrm{i}m^2 \bar{\theta}\gamma^\rho\gamma_5\theta x^\nu M_{\nu\rho}), \end{cases} \tag{$3.14_\pm$}$$

where $x^{\mathrm{L}}_\mu(x, \theta)$, $\theta^{\mathrm{L}}_\alpha(x, \theta)$ and $x^{\mathrm{R}}_\mu(x, \theta)$, $\theta^{\mathrm{R}}_\alpha(x, \theta)$ are just the functions given by relations (3.10) and by those arising from (3.10) after the change $\theta_+ \leftrightarrow \theta_-$, while η^{R} and η^{L} are expressed as

$$\begin{aligned} \eta^{\mathrm{R}}(x, \theta) &= \theta_- + \tfrac{3}{8}\mathrm{i}m^2 \bar{\theta}\theta \,.\, x\gamma\theta_+ = (1 + \tfrac{1}{2}m\bar{\theta}^{\mathrm{L}}\theta^{\mathrm{L}})\theta^{\mathrm{R}} + \tfrac{3}{4}\mathrm{i}m^2 \bar{\theta}^{\mathrm{R}}\theta^{\mathrm{R}}(x^{\mathrm{L}}\gamma)\theta^{\mathrm{L}}, \\ \eta^{\mathrm{L}}(x, \theta) &= \theta_+ + \tfrac{3}{8}\mathrm{i}m^2 \bar{\theta}\theta \,.\, x\gamma\theta_- = (1 + \tfrac{1}{2}m\bar{\theta}^{\mathrm{R}}\theta^{\mathrm{R}})\theta^{\mathrm{L}} + \tfrac{3}{4}\mathrm{i}m^2 \bar{\theta}^{\mathrm{L}}\theta^{\mathrm{L}}(x^{\mathrm{R}}\gamma)\theta^{\mathrm{R}}. \end{aligned} \tag{3.15}$$

In what follows, the bases x^{L}, θ^{L}, η^{R} and x^{R}, θ^{R}, η^{L} associated with the coset parametrisations (3.13_+) and (3.13_-) will be referred to as left- and right-handed, respectively.

It is clear now that x^{L}_μ, $\theta^{\mathrm{L}}_\alpha$ and x^{R}_μ, $\theta^{\mathrm{R}}_\alpha$ are nothing but coordinates of the homogeneous spaces $\mathrm{OSp}(1,4)/\mathrm{S}_-$ and $\mathrm{OSp}(1,4)/\mathrm{S}_+$. The invariance of these superspaces with respect to the realisation of OSp(1,4) as left multiplications of elements G^+ and G^- is now evident and follows directly from the structure of G^+, G^-. It may be verified, in particular, that under the multiplication of G^+ by $G_0 = \mathrm{e}^{\mathrm{i}\bar{\epsilon}Q}$, coordinates x^{L}_μ and $\theta^{\mathrm{L}}_\alpha$ transform according to the law (3.11). The coordinates η^{R} and η^{L} extend the invariant subspaces x^{L}, θ^{L} and x^{R}, θ^{R} to the whole superspace OSp(1,4)/O(1,3) and have a clear meaning: they label 'points' of the purely Grassmannian coset spaces

$S_-/O(1,3)$ and $S_+/O(1,3)$. Under the action of OSp(1,4), the variables η^R, η^L transform according to the left realisations of supergroups S_-, S_+ on these cosets, with constant parameters if an OSp(1,4)-transformation belongs to S_- or S_+ and, otherwise, with parameters dependent on x^L, θ^L or x^R, θ^R, respectively. To illustrate this point we trace in detail how OSp(1,4) operates, say on elements G^+:

$$\begin{aligned} G_0G^+(x^L, \theta^L, \eta^R) &= g(x^{L'}) \exp(i\bar{Q}_+\theta^{L'})S^0_-(G_0, x^L, \theta^L) \exp(i\bar{Q}_-\eta^R) \\ &= g(x^{L'}) \exp(i\bar{Q}_+\theta^{L'}) \exp(i\bar{Q}_-\eta^{R'}) \exp[\tfrac{1}{2}i\tilde{W}^{\mu\nu}_+(G_0, x^L, \theta^L, \eta^R)M_{\mu\nu}] \\ &\equiv G^+(x^{L'}, \theta^{L'}, \eta^{R'}) \exp[\tfrac{1}{2}i\tilde{W}^{\mu\nu}_+(G_0, x^L, \theta^L, \eta^R)M_{\mu\nu}]. \end{aligned} \quad (3.16)$$

Here S^0_- is an element of the right-handed Lorentz supergroup. If $G_0 \in S_-$, then $S^0_-(G_0, x^L, \theta^L) = \text{constant} = G_0$ and $\tilde{W}^{\mu\nu}_+ = \tilde{W}^{\mu\nu}_+(G_0, \eta^R)$.

Further in this section we shall deal only with the parametrisation (3.13_+), keeping in mind that the transition to the basis associated with (3.13_-) can be performed at any stage by means of the trivial interchanges $L \leftrightarrow R$, $\frac{1}{2}(1+i\gamma_5) \leftrightarrow \frac{1}{2}(1-i\gamma_5)$.

Transformation properties of superfields in the left-handed basis are defined in entire analogy to (2.21):

$$\begin{aligned} \tilde{\Phi}_k(x^L, \theta^L, \eta^R) &\xrightarrow{G_0} \tilde{\Phi}'_k(x^{L'}, \theta^{L'}, \eta^{R'}) \\ &= \{\exp[\tfrac{1}{2}i\tilde{W}^{\mu\nu}_+(G_0, x^L, \theta^L, \eta^R)J_{\mu\nu}]\}_{kl}\tilde{\Phi}_l(x^L, \theta^L, \eta^R) \end{aligned} \quad (3.17)$$

where $J_{\mu\nu}$ are, as before, matrices of generators of the Lorentz group (not necessarily constrained by (3.4)). The transformation law (3.17) corresponds to successive inductions: transformations belonging to S_- are induced by the subgroup O(1,3) as a little group (under the action of S_- superfields transform as in O(1,3) but with parameters which are in general functions of η^R and constant parameters of S_-) and the remaining OSp(1,4)-transformations are induced, in turn, by the supergroup S_- (the constant group parameters in the S_- transformation rule are replaced by suitable functions both of x^L, θ^L and new group parameters determined from the composition law (3.16)). The relation of superfields $\tilde{\Phi}_k(x^L, \theta^L, \eta^R)$ to those in the real basis $\Phi_k(x, \theta)$ is not so simple as in the usual supersymmetry:

$$\tilde{\Phi}_k[x^L(x, \theta), \theta^L(x, \theta), \eta^R(x, \theta)] = [\exp(-\tfrac{1}{4}im^2\bar{\theta}\gamma^\rho\gamma_5\theta x^\nu J_{\nu\rho})]_{kl}\Phi_l(x, \theta). \quad (3.18)$$

Here x^L, θ^L, η^R are assumed to be expressed in terms of x, θ through (3.10) and (3.15). Note the presence of the matrix Lorentz factor in (3.18) (which has already appeared in equation (3.9)). It reflects the fact that the coset representatives $G(x, \theta)$ and $G^+(x^L, \theta^L, \eta^R)$, as seen from the relation (3.14_+), are not identical but differ by the Lorentz rotation $\exp(-\frac{1}{4}im^2\bar{\theta}\gamma^\rho\gamma_5\theta x^\nu M_{\nu\rho})$. One can be convinced that the intertwining property of this matrix,

$$\begin{aligned} &\exp(\tfrac{1}{2}i\tilde{W}^{\mu\nu}_+(G_0, x^L, \theta^L, \eta^R)J_{\mu\nu}) \exp(-\tfrac{1}{4}im^2\bar{\theta}\gamma^\rho\gamma_5\theta x^\nu J_{\nu\rho}) \\ &\quad = \exp(-\tfrac{1}{4}im^2\bar{\theta}'\gamma^\rho\gamma_5\theta' x^{\nu'} J_{\nu\rho}) \exp(\tfrac{1}{2}iW^{\mu\nu}(G_0, x, \theta)J_{\mu\nu}), \end{aligned}$$

(which is the consistency condition between realisations (3.16) and (2.20)) guarantees for superfields $\tilde{\Phi}_k$ defined by (3.18) the transformation rule (3.17). The connection (3.18) is, of course, invertible because x_μ and θ_α can always be expressed in terms of x^L_μ, θ^L_α, η^R_α upon inverting equations (3.10) and (3.15).

Covariant derivatives of superfields in the left-handed basis can be found either starting from their expressions (2.28), (2.29) in the symmetric basis and then performing the transformation (3.18) or directly, with the help of the relevant Cartan forms (defined by the decomposition of $(G^+)^{-1}\,dG^+$ in the OSp(1,4)-generators). Without going into details of the derivation, we quote the result:

$$\hat{\mathscr{D}}_+^{\rm R} = (1+\tfrac{1}{4}m\bar{\eta}^{\rm R}\eta^{\rm R})\,\partial/\partial\bar{\eta}^{\rm R} - \tfrac{1}{4}m(\sigma^{\mu\nu}\eta^{\rm R})J_{\mu\nu}, \tag{3.19}$$

$$\hat{\mathscr{D}}_+^{\rm L} = (1+\tfrac{1}{2}m\bar{\eta}^{\rm R}\eta^{\rm R})[(1+\tfrac{1}{4}m\bar{\theta}^{\rm L}\theta^{\rm L})\,\partial/\partial\bar{\theta}^{\rm L} - \tfrac{1}{4}m(\sigma^{\mu\nu}\theta^{\rm L})J_{\mu\nu}] - {\rm i}(\gamma^\nu\eta^{\rm R})\hat{\nabla}^{\rm L}_{+\nu}, \tag{3.20}$$

$$\begin{aligned}\hat{\nabla}^{\rm L}_{+\nu} = \tfrac{1}{2}{\rm i}m[&(1+\tfrac{1}{4}m\bar{\eta}^{\rm R}\eta^{\rm R})(1+\tfrac{1}{4}m\bar{\theta}^{\rm L}\theta^{\rm L})(\bar{\eta}^{\rm R}\gamma_\nu - mx^{\rm L}_\rho\bar{\theta}^{\rm L}\sigma^\rho_\nu)\,\partial/\partial\bar{\theta}^{\rm L}\\ &+(1-\tfrac{1}{4}m\bar{\eta}^{\rm R}\eta^{\rm R})(1-\tfrac{1}{4}m\bar{\theta}^{\rm L}\theta^{\rm L})(\bar{\theta}^{\rm L}\gamma_\nu - mx^{\rm L}_\rho\bar{\eta}^{\rm R}\sigma^\rho_\nu)\,\partial/\partial\bar{\eta}^{\rm R}]\\ &+(1+\tfrac{1}{4}\bar{\eta}^{\rm R}\eta^{\rm R})(1-\tfrac{1}{4}m\bar{\theta}^{\rm L}\theta^{\rm L})[a^{-1}(x^{\rm L})\partial^{\rm L}_\nu - {\rm i}m^2x^{\rm L}_\rho J^\rho_\nu].\end{aligned} \tag{3.21}$$

In the contraction limit expressions (3.19)–(3.21) become covariant derivatives of the usual supersymmetry in the left-handed chiral basis. The form of the derivative $\hat{\mathscr{D}}_+^{\rm R}$ is so simple because it is nothing else than the covariant derivative of the realisation of S_- on the coset space $S_-/O(1,3) = e^{i\bar{Q}_-\eta^{\rm R}}$. Its covariance with respect to the whole supergroup OSp(1,4) follows from the abovementioned fact that the general transformation law (3.17) may be obtained from the transformation law for superfields $\tilde{\Phi}_k(x^{\rm L}, \theta^{\rm L}, \eta^{\rm R})$ in the supergroup S^- (it corresponds to the choice of $\tilde{W}_+^{\mu\nu}(G_0 \in S_-, \eta^{\rm R})$) by changing constant parameters of S_- to certain functions of coordinates $x^{\rm L}$ and $\theta^{\rm L}$ (these functions are still constants relative to $\eta^{\rm R}$-differentiation).

Now it becomes clear what is the origin of the constraints $(3.4_\pm)$. The condition (3.3_+) in the splitting parametrisation takes the form

$$(\hat{\mathscr{D}}_+^{\rm R}\tilde{\Phi}(x^{\rm L}, \theta^{\rm L}, \eta^{\rm R}))_k = 0. \tag{3.22$_+$}$$

Expanding (3.22_+) in powers of $\eta^{\rm R}$ $(\eta^{\rm R}\eta^{\rm R}\eta^{\rm R} = 0)$, it can be observed that for all superfields on which the covariant derivative (3.19) has non-zero matrix part, equation (3.22_+) permits only the trivial solutions $\tilde{\Phi}_k = 0$. If the matrix part is zero equation (3.22_+) goes over to the condition

$$\frac{\partial}{\partial\bar{\eta}^{\rm R}}\tilde{\Phi}_k(x^{\rm L}, \theta^{\rm L}, \eta^{\rm R}) = 0, \tag{3.23}$$

which simply means the absence of $\eta^{\rm R}$-dependence. The only class of the Lorentz group representations on which the second term of (3.19) vanishes is the class (3.4_+). For scalar superfields the matrix part is certainly absent, while for superfields with indices it is nullified due to the presence of projectors $\frac{1}{2}(1+{\rm i}\gamma_5)$ in the generators of representations (3.4_+) (and the algebraic property $[\frac{1}{2}(1-{\rm i}\gamma_5)\sigma^{\mu\nu}]^\beta_\alpha[\frac{1}{2}(1+{\rm i}\gamma_5)\sigma_{\mu\nu}]^\delta_\gamma = 0$). The constraints (3.4_-) have an analogous interpretation (in the basis (3.13_-)).

The restrictions $(3.4_\pm)$ can also be understood based on pure group-theoretical considerations. So far as $x^{\rm L}$, $\theta^{\rm L}$ and $x^{\rm R}$, $\theta^{\rm R}$ are coordinates of the homogeneous spaces OSp(1,4)/S_- and OSp(1,4)/S_+, the superfields dependent only on $x^{\rm L}$, $\theta^{\rm L}$ or on $x^{\rm R}$, $\theta^{\rm R}$ should transform in OSp(1,4) according to representations induced by the corresponding little groups, S_- or S_+. In other words, they should form in external indices linear multiplets of these supergroups. In appendix 2 we list all linear representations of S_- and S_+ realised in spaces of finite-dimensional representations of the Lorentz group. As follows from our analysis, some Lorentz irreducible multiplet is an invariant space of the supergroup $S_-(S_+)$ only provided it transforms in the Lorentz group according to one of the representations (3.4_+) $((3.4_-))$ (the generator $Q_-(Q_+)$ is zero on such a multiplet). In all other cases several different Lorentz multiplets are needed for composing an irreducible linear multiplet of S_- or S_+. In the superfield language, this effectively emerges as appearance of dependence on $\eta^R(\eta^L)$.

It is instructive to compare the situation with that taking place in the usual supersymmetry, chiral representations of which are induced in complex invariant spaces of contracted versions of the supergroups S_-, S_+, namely $S_-(m=0)$, $S_+(m=0)$. In the contraction limit, the RHS of $(2.2_\pm)$ vanishes so that one may set $Q_-(m=0)$ $(Q_+(m=0))$ to zero on any given Lorentz multiplet without conflicting with the structure relations of $S_-(m=0)$ $(S_+(m=0))$ (this multiplet should be complex, otherwise one arrives at the trivial result $Q(m=0)=0$). Thus, any irreducible complex multiplet of the Lorentz group can be taken as an invariant space of the supergroup $S_-(m=0)$ $(S_+(m=0))$ and hence as a carrier of a certain representation of the whole supergroup realised on cosets over $S_-(m=0)$ or $S_+(m=0)$. This is the reason why in the usual supersymmetry chiral superfields with an arbitrary external Lorentz index are permissible.

Note that the purely chiral representations of the conformal superalgebra are also restricted to the classes $(3.4_\pm)$ (Aneva *et al* 1977). This seems natural in the light of the abovementioned property of the conformal superalgebra to be a closure of two superalgebras OSp(1,4) generated by orthogonal combinations of superconformal spinor charges and having a common O(2,3)-subalgebra. Actually, OSp(1,4) plays a crucial role in forming linear representations of the conformal superalgebra: in a forthcoming paper we show that they can all be induced by a simple procedure in invariant spaces of irreducible representations of OSp(1,4).

Now we obtain the OSp(1,4)-invariant integration measures in left- and right-handed chiral superspaces. Performing the changes of variable (3.10) and (3.15) in the measure (2.33) we obtain the invariant measure of the superspace OSp(1,4)/O(1,3) in the parametrisation (3.13_+):

$$\mathscr{D}M = d^4x^L\, d^2\theta^L\, d^2\eta^R\, a^4(x^L)(1+\tfrac{3}{2}m\bar\theta^L\theta^L)(1+\tfrac{1}{2}m\bar\eta^R\eta^R) \qquad (3.24)$$

(this measure could also be obtained straightforwardly as the outer product of the relevant Cartan forms). The invariant measure in the subspace x^L, θ^L may now be found by integrating (3.24) over $(1/m)\, d^2\eta^R$:

$$\mathscr{D}M^L = d^4x^L\, d^2\theta^L\, a^4(x^L)(1+\tfrac{3}{2}m\bar\theta^L\theta^L). \qquad (3.25)$$

The latter coincides with the measure we have derived earlier (Ivanov and Sorin 1979b). The measure in the superspace x^R, θ^R follows from (3.25) through involution:

$$\mathscr{D}M^R = d^4x^R\, d^2\theta^R\, a^4(x^R)(1+\tfrac{3}{2}m\bar\theta^R\theta^R). \qquad (3.26)$$

We conclude this section with several comments concerning the relation of the group structure presented here to the fundamental chiral structure of supergravity revealed recently by Ogievetsky and Sokatchev (1977, 1978a, b).

In the previous section, we have noticed that the realisation of OSp(1,4) in the symmetric basis can be embedded into a very general supergroup consisting both of arbitrary translations of x, θ and gauge Lorentz rotations of external superfield indices. However, the group really relevant to supergravity is in fact much smaller: it is given by a direct product of general coordinate groups in conjugated chiral superspaces x^L, θ^L and x^R, θ^R (Ogievetsky and Sokatchev 1977, 1978a, b). This group is realised so that all local Lorentz rotations appear in the theory not independently but turn out to be induced by transformations of superspace coordinates (like Lorentz rotations in the OSp(1,4)-transformation rules (2.21), (3.17)). The real part of x^{L} and $x^{\mathrm{R}}=(x^{\mathrm{L}})^*$ is identified in the Ogievetsky–Sokatchev approach with the usual space–time coordinate while the imaginary part is postulated to be a function of the remaining variables. The latter is the axial superfield, the fundamental object in terms of which all geometrical characteristics of superspace (supercurvature, supertorsion, etc.) and all transformation laws can be expressed. It is interesting to compare our results deduced following a quite different procedure with this general picture.

Our starting point will be the observation that the realisation of OSp(1,4) given by the rule (3.11) and its conjugate lie in the abovementioned general coordinate groups (just as chiral realisations of usual supersymmetry). With this in mind, we may, step by step, establish the correspondence with the Ogievetsky–Sokatchev formalism. Like them we might choose coordinates $(x^{\mathrm{L}}, \theta^{\mathrm{L}}, \theta^{\mathrm{R}})$, $(x^{\mathrm{R}}, \theta^{\mathrm{R}}, \theta^{\mathrm{L}})$ to represent left- and right-handed splitting bases instead of using for this purpose coordinate sets $(x^{\mathrm{L}}, \theta^{\mathrm{L}}, \eta^{\mathrm{R}})$, $(x^{\mathrm{R}}, \theta^{\mathrm{R}}, \eta^{\mathrm{L}})$. Indeed, equation (3.15) indicates that η^{R}, η^{L} are canonically related to θ^{R}, θ^{L}. Likewise, we might associate with the symmetric basis the coordinates $\tilde{x}_\mu=\frac{1}{2}(x^{\mathrm{L}}_\mu+x^{\mathrm{R}}_\mu)$, $\tilde{\theta}=\theta^{\mathrm{L}}+\theta^{\mathrm{R}}$ instead of x_μ, θ_α. The explicit form of the equivalence mapping between these two sets of variables can immediately be found with the help of the relations (3.10) and their right-handed counterparts:

$$\begin{aligned}\tilde{x}_\mu&=[1+(m^2/16a(x))(\bar{\theta}\theta)^2]x_\mu,\\ \tilde{\theta}&=\theta-\tfrac{1}{2}m\bar{\theta}\theta(1+\tfrac{3}{4}\mathrm{i}mx\gamma)\theta.\end{aligned} \tag{3.27}$$

In terms of $\tilde{x}_\mu$, $\tilde{\theta}_\alpha$ the transitions to the shifted bases are extremely simple and have almost the same form as in the usual supersymmetry:

$$\begin{pmatrix}x^{\mathrm{L}}_\mu\\ x^{\mathrm{R}}_\mu\end{pmatrix}=\begin{pmatrix}\tilde{x}_\mu-\frac{1}{4}\bar{\tilde{\theta}}\gamma_\mu\gamma_5\tilde{\theta}a^{-1}(\tilde{x})\\ \tilde{x}_\mu+\frac{1}{4}\bar{\tilde{\theta}}\gamma_\mu\gamma_5\tilde{\theta}a^{-1}(\tilde{x})\end{pmatrix},\qquad \begin{pmatrix}\theta^{\mathrm{L}}\\ \theta^{\mathrm{R}}\end{pmatrix}=\begin{pmatrix}\tilde{\theta}_+\\ \tilde{\theta}_-\end{pmatrix}. \tag{3.28}$$

So, only the shift of the boson coordinate is essential and unremovable whereas shifts of Grassmann variables can be absorbed into the equivalence redefinition of coordinates. This agrees with the basic concepts of Ogivetsky and Sokatchev†. It is clear now what plays the role of 'axial superfield' for the superspace OSp(1,4)/O(1,3), namely, $(1/2\mathrm{i})(x^{\mathrm{L}}_\mu-x^{\mathrm{R}}_\mu)=\frac{1}{4}\mathrm{i}\bar{\tilde{\theta}}\gamma_\mu\gamma_5\tilde{\theta}a^{-1}(\tilde{x})=\frac{1}{4}\mathrm{i}\bar{\theta}\gamma_\mu\gamma_5\theta a^{-1}(x)$. It should be expected that all OSp(1,4)-covariant objects and OSp(1,4)-transformation rules found in the present paper admit re-expression in terms of this fundamental geometrical characteristic (and its derivatives), similarly to the pure anti-de Sitter case, where all relevant quantities can be rewritten through $a(x)$ and $\partial_\mu a(x)$.

Finally, we recall that the group of pure Einstein supergravity is restricted by the condition of conservation of volumes of chiral superspaces while the case without any restrictions corresponds to Weyl (conformal) supergravity (Ogievetsky and Sokatchev 1978a, b). The realisation (3.11) and its conjugate certainly do not preserve volume elements $\mathrm{d}^4x^{\mathrm{L}}\,\mathrm{d}^2\theta^{\mathrm{L}}$ and $\mathrm{d}^4x^{\mathrm{R}}\,\mathrm{d}^2\theta^{\mathrm{R}}$ and hence do not belong to the Einstein supergravity subgroup. It can be shown, however, that in the Weyl supergravity group there exists another infinite-parameter subgroup which preserves the OSp(1,4)-invariant volume

† We suspect that coordinates x_μ, θ_α (which are both shifted when transitions to chiral bases are performed) correspond to Siegel's formulation of superspace supergravity (see e.g. Siegel and Gates 1979). This formulation seems to be basically equivalent to the Ogievetsky–Sokatchev approach.

elements (3.25) and (3.26) instead of the usual ones. This group includes as the 'flattest' graded subgroup just OSp(1,4) and therefore seems to be relevant to OSp(1,4)-supergravity.

4. Some OSp(1,4)-invariant models

We construct now OSp(1,4)-analogues of the simplest theories with global Poincaré supersymmetry: the Wess–Zumino model (Wess and Zumino 1974a, b) and the supersymmetric Yang–Mills theory (Ferrara and Zumino 1974, Salam and Strathdee 1974).

4.1. The Wess–Zumino model in anti-de Sitter space

The OSp(1,4)-invariant superfield action for the self-interacting scalar OSp(1,4)-multiplet with overall mass M and dimensionless coupling constant g can be written as

$$
\begin{aligned}
S = S_K + S_M + S_g = \int \mathscr{D}M\, \Phi_+(x,\theta)\Phi_-(x,\theta) \\
+\tfrac{1}{2}M\Big[\int \mathscr{D}M^{\mathrm{L}} T^{+2}(x^{\mathrm{L}},\theta^{\mathrm{L}}) + \int \mathscr{D}M^{\mathrm{R}} T^{-2}(x^{\mathrm{R}},\theta^{\mathrm{R}})\Big] \\
+(\sqrt{2}/3)g\Big[\int \mathscr{D}M^{\mathrm{L}} T^{+3}(x^{\mathrm{L}},\theta^{\mathrm{L}}) + \int \mathscr{D}M^{\mathrm{R}} T^{-3}(x^{\mathrm{R}},\theta^{\mathrm{R}})\Big].
\end{aligned} \tag{4.1}
$$

The invariant integration measures $\mathscr{D}M$, $\mathscr{D}M^{\mathrm{L}}$ and $\mathscr{D}M^{\mathrm{R}}$ are defined by formulae (2.33), (3.25) and (3.26). Chiral superfields in the symmetric basis, Φ_+ and $\Phi_- = (\Phi_+)^*$ are related to 'truncated' superfields T^+ and $T^- = (T^+)^*$ as in (3.6$_\pm$).

Using the rules for changing variables in Grassmann integrals and the connection (3.10) together with its conjugate, we have checked that the second and third pieces in (4.1) do not change their form under the replacements $x^{\mathrm{L}}_\mu, x^{\mathrm{R}}_\mu \to x_\mu$, $\theta^{\mathrm{L}} \to \theta_+$, $\theta^{\mathrm{R}} \to \theta_-$. Then, integrating (4.1) over $\mathrm{d}\theta$, going to real components A, B, F, G,

$$
A_\pm = \frac{1}{\sqrt{2}}(A \pm \mathrm{i}B), \qquad F_\pm = \frac{1}{\sqrt{2}}(F \pm \mathrm{i}G),
$$

and finally eliminating the auxiliary fields F and G by their equations of motion

$$
F = -(m+M)A - g(A^2 - B^2), \qquad G = (M-m)B + 2gAB, \tag{4.2}
$$

we are left with the action which includes only the physical components $\psi(x)$, $A(x)$, $B(x)$ and is expressed solely in terms of anti-de Sitter space:

$$
S = \int \mathscr{D}M_{\mathrm{s}}[\tfrac{1}{2}(\nabla^\mu A \nabla_\mu A + \nabla^\mu B \nabla_\mu B + \mathrm{i}\bar{\psi}\gamma^\mu \nabla_\mu \psi) - V_M(A, B, \psi)] \tag{4.3}
$$

Here ∇_μ and $\mathscr{D}M_{\mathrm{s}}$ are, respectively, the O(2,3)-covariant derivative and O(2,3)-invariant integration measure for anti-de Sitter space given by (2.14) and (2.17) (in fact, the matrix part of $\nabla_\mu \psi$ makes no contribution to (4.3) because of the Majorana spinor property $\bar{\psi}(x)\gamma_\mu\psi(x)=0$). The potential $V_M(A, B, \psi)$ has the form

$$V_M(A, B, \psi)=\tfrac{1}{2}(M+m)(M-2m)A^2+\tfrac{1}{2}(M-m)(M+2m)B^2+\tfrac{1}{2}M\bar{\psi}\psi$$
$$+\tfrac{1}{2}g^2(A^2+B^2)^2+gMA(A^2+B^2)+g\bar{\psi}(A-B\gamma_5)\psi. \tag{4.4}$$

Expressions (4.3) and (4.4) coincide with those we have obtained earlier by a different method (Ivanov and Sorin 1979b). In the contraction limit they reduce to the standard Wess–Zumino (1974a, b) action.

Now we schematically review the main properties of the present model, following our work (Ivanov and Sorin 1979b).

The first thing one observes is that the fermion and boson masses in the potential (4.4) have split already at the level of unbroken OSp(1,4)-symmetry, m being a parameter of splitting. This is to be compared with the situation in the standard Wess–Zumino model where the fermion and both bosons have equal masses. The reason why the OSp(1,4)-symmetry is less restrictive than the usual supersymmetry is quite clear: it is the appearance of the new dimensional constant, m, which non-trivially enters into the Lagrangians and plays, to a certain extent, the role of a free parameter. In view of this, it is not so surprising that, depending on a relation between parameters m and M, the model under consideration realises regimes with a different symmetry of the ground state, whereas its Minkowski prototype displays only a fully symmetric phase. For instance, in the ranges $0\leqslant M\leqslant 2m$ and $-4m\leqslant M\leqslant -2m$ ($m>0$ fixed) $V_M(A, B, \psi)$ has a minimum for the constant solutions

$$\langle A\rangle=\frac{2m-M}{2g}, \qquad \langle B\rangle=\langle\psi\rangle=\langle G\rangle=0, \qquad \langle F\rangle=\frac{(M+4m)(M-2m)}{4g}, \tag{4.5}$$

which gives rise to spontaneous breakdown of OSp(1,4)-supersymmetry to O(2,3)-symmetry. Indeed, O(2,3) is the maximal invariance group of (4.5). OSp(1,4)-supertranslations displace the spinor component $\langle\delta_Q\psi\rangle=(1/\sqrt{2})\langle F\rangle\beta\neq 0$, indicating that in the sector raised upon the minimum (4.5) $\psi(x)$ plays the role of Goldstino. Its mass is $m_\psi=2m$ in complete agreement with the general theorem of Zumino (1977). The masses of boson fields shifted to zero vacuum expectation values are real (i.e. ghosts do not appear) and are related to m_ψ by the simple formula

$$m_A^2+m_B^2=m_\psi^2, \tag{4.6}$$

An unexpected feature of the present model is the existence of the phase with spontaneously broken *P*- and *CP*-parities. In the range $-2m\leqslant M\leqslant 0$ the potential (4.4) attains two absolute minima at the following vacuum expectation values of the fields:

$$\langle A\rangle=\frac{m-M}{2g}, \qquad \langle B\rangle=\pm\frac{[(m-M)(3m+M)]^{1/2}}{2g}, \qquad \langle\psi\rangle=\langle F\rangle=\langle G\rangle=0. \tag{4.7}$$

Upon passing to fields A', B', ψ' which have zero vacuum expectation values and diagonalise the mass matrix, there appears in the rearranged potential the *P*- and (simultaneously) *CP*-violating interaction $\sim\bar{\psi}'(B'+A'\gamma_5)\psi'$, with the constant $\pm g[(M'-m)/2M']^{1/2}$ where M' is the physical mass of ψ' ($2m\leqslant M'\leqslant\sqrt{5}m$). Parameters m, g, M' may be adjusted so that the constant of *CP*-violation is of a reasonable order of magnitude. Again, all the physical masses are strictly real. A more detailed analysis of the *CP*-breaking phase will be given elsewhere. Note that OSp(1,4)-symmetry in this sector is not broken as $\langle F_\pm\rangle = 0$ there.

With any other relations between m and M the symmetric phase is realised in the model. As expected, it is the only phase which survives in the limit $m \to 0$, $M \neq 0$ corresponding to the usual massive Wess–Zumino model.

In the special case $M = 0$, the action (4.1) possesses a wider, superconformal symmetry and, in particular, chiral invariance (with the identification $Q = (1/\sqrt{2})(S + mT)$, $R_\mu = \frac{1}{2}(P_\mu - m^2 K_\mu)$, where S, T, P_μ and K_μ are generators of the conformal supergroup normalised as in Ivanov and Sorin (1979a)). The minima of the potential $V_{M=0}(A, B, \psi)$ are placed on the circle of radius $|m/g|$ in the A–B plane, solutions (4.5) and (4.7) going into certain points on that circle. The violation of *P*- and *CP*-parities becomes unobservable as the relevant sector turns out to be related to *P*- and *CP*-preserving sectors by a chiral transformation of fields. Each minimum is invariant under a certain subgroup OSp(1,4) of the conformal supergroup and realises the spontaneous breakdown of symmetry with respect to another graded OSp(1,4)-subgroup which has the same O(2,3)-subgroup but whose odd generator is given by an orthogonal combination of superconformal spinor charges (for more details see Ivanov and Sorin (1979b)).

At $M = 0$ the action (4.3) exhibits also the generalised Weyl covariance. To be more precise, it is brought into the ordinary superfield action of the usual massless Wess–Zumino model by the superfield Weyl transformation

$$T^\pm(x, \theta_\pm) \to T^{\pm\prime}(x, \theta_\pm) = a(x)[1 + \tfrac{1}{2} m a(x) \bar{\theta}_\pm \theta_\pm] T^\pm[x, (a(x))^{1/2}\theta_\pm] \qquad (4.8)$$

(Ivanov and Sorin 1979b). For physical components the transformation (4.8) is the standard Weyl transformation from anti-de Sitter space to Minkowski space:

$$A'(x) = a(x)A(x), \qquad B'(x) = a(x)B(x), \qquad \psi'(x) = a^{3/2}(x)\psi(x) \qquad (4.9)$$

(see e.g. Fronsdal 1975, Domokos 1976). It can straightforwardly be verified that in terms of the fields A', B' and ψ' the $M = 0$ limit of the action (4.3) coincides with the component action of the massless Wess–Zumino model in Minkowski space. The constant solutions giving minima to the potential $V_{M=0}(A, B, \psi)$ transform into x-dependent classical solutions of the Fubini (1976) type discussed by Baaklini (1977) and Ivanov and Sorin (1979a). Note that (4.9) is a particular case of the transformation to a chiral superfield of superconformal weight n defined in appendix 3 (it corresponds to $n = \frac{1}{2}$).

We have confined our study here to purely classical considerations. In principle, one may take the present model more seriously and try to allow for quantum loop corrections. This can be done using the quantisation schemes for anti-de Sitter space suggested by Fronsdal (1975 and references therein) and Avis *et al* (1978).

4.2. OSp (1,4)-extension of the Yang–Mills theory

OSp(1,4)-invariant gauge theories are constructed analogously to the usual supersymmetric ones. We prefer to work in the left-handed basis (3.13_+). Let I_i be the matrices of the generators of some group of internal symmetry G. Introduce the real Lie-algebra-valued gauge superfield

$$V(x^{\mathrm{L}}, \theta^{\mathrm{L}}, \eta^{\mathrm{R}}) = V^*(x^{\mathrm{L}}, \theta^{\mathrm{L}}, \eta^{\mathrm{R}}) = V^i(x^{\mathrm{L}}, \theta^{\mathrm{L}}, \eta^{\mathrm{R}}) I_i \qquad (4.10)$$

and postulate for it the following law of local G-transformations:

$$\exp(2gV'(x^{\mathrm{L}}, \theta^{\mathrm{L}}, \eta^{\mathrm{R}})) = \exp(-\mathrm{i}\Lambda^{+}(x^{\mathrm{L}}, \theta^{\mathrm{L}}, \eta^{\mathrm{R}}))\exp(2gV(x^{\mathrm{L}}, \theta^{\mathrm{L}}, \eta^{\mathrm{R}}))\exp(\mathrm{i}\Lambda(x^{\mathrm{L}}, \theta^{\mathrm{L}})) \tag{4.11}$$

where g is a constant and Λ, Λ^{+} are two conjugated superfunctions with values in the same algebra, and subject to the conditions

$$\hat{\mathscr{D}}^{\mathrm{R}}_{+}\Lambda = \hat{\mathscr{D}}^{\mathrm{L}}_{+}\Lambda^{+} = 0 \tag{4.12}$$

($\hat{\mathscr{D}}^{\mathrm{R}}_{+}$ and $\hat{\mathscr{D}}^{\mathrm{L}}_{+}$ are defined by formulae (3.19) and (3.20)). Then the left-handed (with respect to the Lorentz index) spinor superfield $\mathrm{e}^{-2gV}\hat{\mathscr{D}}^{\mathrm{L}}_{+}\,\mathrm{e}^{2gV}$ transforms in G as

$$\mathrm{e}^{-2gV}\hat{\mathscr{D}}^{\mathrm{L}}_{+}\,\mathrm{e}^{2gV} \rightarrow \mathrm{e}^{-\mathrm{i}\Lambda}\,\mathrm{e}^{-2gV}\hat{\mathscr{D}}^{\mathrm{L}}_{+}\,\mathrm{e}^{2gV}\,\mathrm{e}^{\mathrm{i}\Lambda} + \mathrm{e}^{-\mathrm{i}\Lambda}\hat{\mathscr{D}}^{\mathrm{L}}_{+}\,\mathrm{e}^{\mathrm{i}\Lambda}. \tag{4.13}$$

A complication in comparison with the usual supersymmetry arises only at the stage of construction of covariant superfield strengths. The operator $\bar{\hat{\mathscr{D}}}^{\mathrm{R}}_{+}\hat{\mathscr{D}}^{\mathrm{R}}_{+}$ being applied to the superfield (4.13) does not produce the covariant quantity because in the OSp(1,4)-case it does not annihilate the inhomogeneous term in (4.13). The correct generalisation of the standard procedure implies the use of the projection operator

$$\Pi^{(1/2,0)}_{-} = -(1/2m)(\bar{\hat{\mathscr{D}}}^{\mathrm{R}}_{+}\hat{\mathscr{D}}^{\mathrm{R}}_{+} - 2m) \tag{4.14}$$

which singles out from the left-handed spinor OSp(1,4)-superfield the pure chiral part dependent only on x^{L} and θ^{L} (see appendix 2). As is shown in appendix 2, such a superfield divides in general into two pieces each closed under the action of OSp(1,4). One of them is a pure chiral superfield, the other a non-chiral superfield, F- and A-components in the expansion of which in η^{R} are related as $F_{\alpha}(x^{\mathrm{L}}, \theta^{\mathrm{L}}) = -mA_{\alpha}(x^{\mathrm{L}}, \theta^{\mathrm{L}})$ (formulae (A2.15), (A2.14)). Making use of the explicit form (3.20) for the covariant derivative $\hat{\mathscr{D}}^{\mathrm{L}}_{+}$, it can be established that the inhomogeneous term in (4.13) is just a superfield of the second type. Therefore the action of the projection operator (4.14) on that term gives zero†. As a result, the left-handed chiral spinor superfield

$$W_{+\alpha} = -[2m\Pi^{(1/2,0)}_{-}\,\mathrm{e}^{-2gV}\hat{\mathscr{D}}^{\mathrm{L}}_{+}\,\mathrm{e}^{2gV}]_{\alpha} = (\bar{\hat{\mathscr{D}}}^{\mathrm{R}}_{+}\hat{\mathscr{D}}^{\mathrm{R}}_{+} - 2m)^{\beta}_{\alpha}\,\mathrm{e}^{-2gV}\hat{\mathscr{D}}^{\mathrm{L}}_{+\beta}\,\mathrm{e}^{2gV} \tag{4.15}$$

transforms in G homogeneously,

$$W_{+\alpha} \rightarrow \mathrm{e}^{-\mathrm{i}\Lambda}W_{+\alpha}\,\mathrm{e}^{\mathrm{i}\Lambda}, \tag{4.16}$$

and can serve as the covariant strength. In the limit $m \rightarrow 0$ it becomes the covariant superfield strength of the conventional supersymmetric Yang–Mills theory.

The invariant action for the gauge superfield is set up in the standard manner,

$$S = \frac{1}{g^2}\int \mathscr{D}M^{\mathrm{L}}\,\mathrm{Tr}(\bar{W}_{+}W_{+}) + \mathrm{HC}, \tag{4.17}$$

and in the Wess–Zumino gauge has the form

$$S = \int \mathrm{d}^4x\, a^4(x)\,\mathrm{Tr}[-\tfrac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} + \tfrac{1}{2}\mathrm{i}\bar{\lambda}\gamma^{\mu}\tilde{\mathscr{D}}_{\mu}\lambda + \tfrac{1}{2}D^2], \tag{4.18}$$

where

$$\tilde{F}_{\mu\nu} = \nabla_{\mu}v_{\nu} - \nabla_{\nu}v_{\mu} + \mathrm{i}g[v_{\mu}, v_{\nu}], \tag{4.19}$$

$$\tilde{\mathscr{D}}_{\mu}\lambda = \nabla_{\mu}\lambda + \mathrm{i}g[v_{\mu}, \lambda]. \tag{4.20}$$

† This fact can be verified straightforwardly, using the algebra of covariant derivatives (2.30).

The action (4.18) describes the Yang–Mills field v^i_μ in anti-de Sitter space minimally coupled to the massless Majorana spinor λ^i_α belonging to the regular representation of the group G. Transformations of OSp(1,4)-symmetry in the Wess-Zumino gauge are as follows:

$$\begin{aligned} \delta v_\mu &= \mathrm{i}\bar{\beta}\gamma_\mu\lambda, \qquad \delta D = \mathrm{i}\bar{\beta}\gamma^\mu\gamma_5\tilde{\mathscr{D}}_\mu\lambda, \\ \delta\lambda &= -\tfrac{1}{2}\mathrm{i}\sigma^{\mu\nu}\beta\tilde{F}_{\mu\nu} + \gamma_5\beta D, \end{aligned} \tag{4.21}$$

where β is the same function as in (3.12).

We emphasise that the action (4.18) in itself produces no new consequences since upon the Weyl transformation

$$v_\mu(x), \lambda(x), D(x) \to v'_\mu(x) = a(x)v_\mu(x), \lambda'(x) = a^{3/2}(x)\lambda(x), D'(x) = a^2(x)D(x) \tag{4.22}$$

it reduces to the ordinary action of the corresponding supersymmetric Yang–Mills theory in Minkowski space. This is because the action (4.18) is conformally invariant and, hence, Weyl-covariant. Moreover, the component transformations (4.22) can be extended to the Weyl superfield transformation which takes the action (4.17) into the superfield action of the related gauge theory in the usual superspace (for brevity, we do not give it explicitly; it is similar to (4.8)). This reflects the generalised Weyl covariance of supersymmetric Yang–Mills theories caused by their superconformal invariance.

A non-trivial novel theory may be set up, e.g. by coupling the gauge superfield to the massive scalar OSp(1,4)-multiplet belonging to a unitary representation of group G (such interactions are introduced in the same way as in the usual supersymmetry). Models of this type possess no superconformal invariance (and Weyl covariance) and therefore allow no transition to Minkowski space. We intend to explore them in the future.

For completeness, we finally quote general expressions for component Lagrangian densities belonging to scalar and vector OSp(1,4)-multiplets:

$$\mathscr{L}_{\mathrm{I}}(x) = a^4(x)(F(x) + 3mA(x)), \tag{4.23}$$

$$\mathscr{L}_{\mathrm{II}}(x) = a^4(x)(D(x) + 12m^2A(x) + 12mF(x)). \tag{4.24}$$

$\mathscr{L}_{\mathrm{I}}$ and $\mathscr{L}_{\mathrm{II}}$ have positive parity and change by a divergence under OSp(1,4)-transformations.

5. Conclusion

In this paper we have described the superfield approach to supersymmetry in anti-de Sitter space and constructed the simple linear globally OSp(1,4)-invariant models. The most intriguing possibility provided by OSp(1,4)-supersymmetry is, in our opinion, a natural emergence of spontaneous breakdown of P- and CP-parities with the breaking parameter related to radius of anti-de Sitter space. It may be expected that this phenomenon we have revealed in the simplest OSp(1,4)-invariant model will be retained in more complicated theories including those with local OSp(1,4)-symmetry. We also hope that the methods elaborated here can be generalised to cover the case of OSp(N,4)-supersymmetry. The construction and study of models with global OSp(N,4)-symmetry is an interesting and urgent task since such models describe the 'flat' limit of O(N)-extended supergravity, a self-contained superfield formulation of which is as yet unknown.

Our approach is based on the consistent application of group-theoretical methods. However, as has been already partly explained in the text, all the relevant relations could in principle be obtained by a limiting procedure from more general local theories. For instance, the basic elements of O(2,3)-formalism constructed in § 2.2 can be deduced in a different manner, by noting that the space O(2,3)/O(1,3) is a particular solution of Einstein's equations with a negative cosmological term and substituting the relevant background metric into general relativity formulae. Likewise, the superspace OSp(1,4)/O(1,3) (as well as OSp(1,4)/S_- and OSp(1,4)/S_+) is expected to be a particular solution of equations of superfield supergravity (e.g. in the Wess–Zumino (1977, 1978), Ogievetsky–Sokatchev (1977, 1978a, b) or Siegel–Gates (1979) formulations). Therefore, all relations of OSp(1,4)-covariant formalism should result from general formulae of a self-contained superfield supergravity on inserting the corresponding particular values of gauge superfields. To verify this, we might proceed, say, from the explicit form of the OSp(1,4)-solution for the Ogievetsky–Sokatchev axial superfield found in § 3.3. But, as a closed formulation of OSp(1,4)-supergravity (as well as of a conformal one) in terms of the axial superfield (an invariant action, superfield equations of motion etc.) has not been constructed for the time being, it is simpler to work within component supergravity (Stelle and West 1978, Ferrara and Van Nieuwenhuizen 1978b, Ferrara *et al* 1978) which seems to be equivalent to a certain gauge of the complete superfield theory. A particular set of gauge fields of component supergravity related to the superspace OSp(1,4)/O(1,3) includes (in our notation) the O(2,3)-de Sitter solution for the vierbein $e^{\alpha}_{\mu} = \delta^{\alpha}_{\mu} a(x)$ and the constant solution for the auxiliary field $S = 3\sqrt{2}m$ (all other gauge fields, ψ_{μ}, A_{μ}, P, take zero values). Inserting this set for example in the general transformation law for a scalar multiplet of local supersymmetry (Ferrara and Van Niewenhuizen 1978a) with the restriction to OSp(1,4)-transformations only, one reproduces the transformation law (3.12). Analogously, starting from general couplings of a scalar multiplet with supergravity (Cremmer *et al* 1979), it is possible to regain the action considered in § 4.1. The advantage of the approach we have developed is that it allows one to deduce all the relations of OSp(1,4)-supersymmetry on the basis of the OSp(1,4)-superalgebra (2.1) alone and provides a deep understanding of the group structure of this important second global limit of local supersymmetry. Moreover, it may serve as a useful guide in constructing a closed superfield formulation of OSp(1,4)-supergravity.

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Appendix 1. OSp(1,4)-generators in nonsymmetric parametrisation

Taking into account that the left- and right-handed splitting bases (3.13_+), (3.13_-) are related through involution, it is sufficient to know expressions for generators in one of them, say in the basis (3.13_+):

$$M_{\mu\nu} = \mathrm{i}(x^{\mathrm{L}}_\mu \partial^{\mathrm{L}}_\nu - x^{\mathrm{L}}_\nu \partial^{\mathrm{L}}_\mu) + \tfrac{1}{2}\bar{\theta}^{\mathrm{L}}\sigma_{\mu\nu}\,\partial/\partial\bar{\theta}^{\mathrm{L}} + M^{S_-}_{\mu\nu}, \tag{A1.1}$$

$$Q = -a^{-1}(x^{\mathrm{L}})\Lambda(x^{\mathrm{L}})\gamma^\mu\theta^{\mathrm{L}}\partial^{\mathrm{L}}_\mu + \mathrm{i}\Lambda(x^{\mathrm{L}})[1-\tfrac{1}{2}m\bar{\theta}^{\mathrm{L}}\theta^{\mathrm{L}}(1-\tfrac{3}{2}\mathrm{i}mx^{\mathrm{L}}_\mu\gamma^\mu)]\,\partial/\partial\bar{\theta}^{\mathrm{L}}$$
$$+(1+\tfrac{1}{2}m\bar{\theta}^{\mathrm{L}}\theta^{\mathrm{L}})\Lambda(x^{\mathrm{L}})Q^{S_-} + \tfrac{1}{4}\mathrm{i}m\Lambda(x^{\mathrm{L}})(\sigma_{\mu\nu}+4mx^{\mathrm{L}}_\mu\gamma_\nu)\theta^{\mathrm{L}}M^{S_-\mu\nu}, \tag{A1.2}$$

$$R_\mu = \mathrm{i}\left(\frac{1-m^2x^{\mathrm{L}2}}{2}\delta^\nu_\mu + m^2x^{\mathrm{L}}_\mu x^{\mathrm{L}\nu}\right)\partial^{\mathrm{L}}_\nu - \frac{m^2}{2}x^{\mathrm{L}}_\nu\bar{\theta}^{\mathrm{L}}\sigma^\nu_\mu\frac{\partial}{\partial\bar{\theta}^{\mathrm{L}}} + m^2x^{\mathrm{L}\nu}M^{S_-}_{\mu\nu}. \tag{A1.3}$$

Here

$$M^{S_-}_{\mu\nu} = \tfrac{1}{2}\bar{\eta}^{\mathrm{R}}\sigma_{\mu\nu}\,\partial/\partial\bar{\eta}^{\mathrm{R}} + J_{\mu\nu}, \tag{A1.4}$$

$$Q^{S_-} = \mathrm{i}(1-\tfrac{1}{2}m\bar{\eta}^{\mathrm{R}}\eta^{\mathrm{R}})\,\partial/\partial\bar{\eta}^{\mathrm{R}} + \tfrac{1}{4}\mathrm{i}m\sigma^{\mu\nu}\eta^{\mathrm{R}}J_{\mu\nu}, \tag{A1.5}$$

are generators of the supergroup S_- realised in the coset space $S_-/O(1,3)$.

Appendix 2. Irreducible spaces of the supergroups S_- and S_+

In § 3.3 we have remarked that linear representations of the supergroup OSp(1,4) in the parametrisations (3.13_+) and (3.13_-) can be regarded as induced in invariant spaces of its supersubgroups S_-, S_+ respectively (with O(1,3) as the structure group). In other words, given, for example, a space closed under the action of S_-†:

$$\Phi_k(\eta^{\mathrm{R}}) = P_k + \bar{\eta}^{\mathrm{R}}N_k + \bar{\eta}^{\mathrm{R}}\eta^{\mathrm{R}}B_k \tag{A2.1}$$

(where k is the external Lorentz index), one immediately arrives at the space invariant under the whole supergroup OSp(1,4) (and hence carrying its linear representation), simply by replacing constant coefficients in (A2.1) by functions over the homogeneous space OSp(1,4)/S_-:

$$P_k, N_k, B_k \to P_k(x^{\mathrm{L}}, \theta^{\mathrm{L}}), N_k(x^{\mathrm{L}}, \theta^{\mathrm{L}}), B_k(x^{\mathrm{L}}, \theta^{\mathrm{L}}). \tag{A2.2}$$

So, the problem of implementing all inequivalent linear representations of OSp(1,4) in the left-handed basis (3.13_+) reduces to finding all inequivalent irreducible invariant spaces of the supergroup S_- of the type (A2.1) (the analogous statement, with the change $S_-\to S_+$, holds also for the right-handed basis). Henceforth, we restrict our consideration to the case of S_-, keeping in mind that invariant spaces of S_+ can be obtained from those of S_- by involution (just as S_+ itself can be obtained from S_-).

To obtain all the irreducible S_--invariant spaces of the type (A2.1) it is sufficient to find the spectrum of two Casimir operators of S_-:

$$\hat{C}_1 = M^{S_-}_{\mu\nu}M^{S_-\mu\nu} - (1/m)\bar{Q}^{S_-}Q^{S_-}, \qquad \hat{C}_2 = M^{S_-}_{\mu\nu}M^{S_-\mu\nu} + \tfrac{1}{2}\mathrm{i}\epsilon^{\mu\nu\rho\lambda}M^{S_-}_{\mu\nu}M^{S_-}_{\rho\lambda}, \tag{A2.3}$$

where $M^{S_-}_{\mu\nu}$, Q^{S_-} are given by (A1.4) and (A1.5). Inserting in (A2.3) the explicit forms for the generators and using (3.19), we obtain:

$$(\hat{C}_1)^l_k = (1/m)(\hat{\bar{\mathscr{D}}}{}^{\mathrm{R}}_+\hat{\mathscr{D}}^{\mathrm{R}}_+)^l_k + (\hat{K}_1)^l_k, \qquad (\hat{C}_2)^l_k = (\hat{K}_1+\hat{K}_2)^l_k \tag{A2.4}$$

where $(\hat{\bar{\mathscr{D}}}{}^{\mathrm{R}}_+\hat{\mathscr{D}}^{\mathrm{R}}_+)^l_k = (\hat{\bar{\mathscr{D}}}{}^{\mathrm{R}\beta}_+)^{\gamma m}_{\beta k}(\hat{\mathscr{D}}^{\mathrm{R}}_{+\gamma})^l_m$ and $\hat{K}_1$, $\hat{K}_2$ are purely matrix parts of the Casimir operators of the Lorentz group:

$$\hat{K}_1 = J_{\mu\nu}J^{\mu\nu}, \qquad \hat{K}_2 = \tfrac{1}{2}\mathrm{i}\epsilon^{\mu\nu\rho\lambda}J_{\mu\nu}J_{\rho\lambda}. \tag{A2.5}$$

† We apply here the terms 'invariant space' and 'superfield' on an equal footing.

We suppose that external indices of the superfields (A2.1) are rotated by the finite-dimensional irreducible representations of the Lorentz group $D^{(p,q)}$, where p and q are positive integers and half-integers. For such representations the spectrum of the operators (A2.5) has the form

$$\hat{K}_1^{(p,q)} = 4p(p+1) + 4q(q+1), \qquad \hat{K}_2^{(p,q)} = 4p(p+1) - 4q(q+1). \tag{A2.6}$$

Now, the use of the simple identity for the covariant derivative $\hat{\mathscr{D}}_+^{\mathrm{R}}$,

$$(\bar{\hat{\mathscr{D}}}{}_+^{\mathrm{R}} \hat{\mathscr{D}}_+^{\mathrm{R}})^2 = \tfrac{1}{2}m^2(\hat{K}_1 - \hat{K}_2) + 2m\bar{\hat{\mathscr{D}}}{}_+^{\mathrm{R}} \hat{\mathscr{D}}_+^{\mathrm{R}},$$

yields the spectrum of the operator $\hat{C}_1 - \hat{K}_1$:

$$(\hat{C}_1 - \hat{K}_1)_{(\pm)}^{(p,q)} = 1 \pm (1+2q). \tag{A2.7}$$

Finally, the total spectrum of the operators $\hat{C}_1$ and $\hat{C}_2$ on superfields (A2.1) is given by

$$\hat{C}_{1(\pm)}^{(p,q)} = 4p(p+1) + 4q(q+1) + 1 \pm (1+2q), \qquad \hat{C}_2^{(p,q)} = 8p(p+1). \tag{A2.8}$$

It is seen that the eigenvalue of $\hat{C}_2$ is uniquely determined by the superfield external index, k. At the same time, to each fixed q there correspond two different eigenvalues of $\hat{C}_1$ distinguished by signs $(+, -)$. Consequently, each superfield (A2.1) with the fixed Lorentz index (i.e. p and q fixed) contains two irreducible inequivalent subspaces of the supergroup S_-. The normalised projection operators which single out these subspaces are constructed in a standard manner:

$$\begin{gathered}\Pi_{\pm}^{(p,q)} = \frac{\hat{C}_1 - \hat{C}_{1(\mp)}^{(p,q)}}{\hat{C}_{1(\pm)}^{(p,q)} - \hat{C}_{1(\mp)}^{(p,q)}} = \pm \frac{\bar{\hat{\mathscr{D}}}{}_+^{\mathrm{R}} \hat{\mathscr{D}}_+^{\mathrm{R}} - [1 \mp (1+2q)]m}{2m(1+2q)} \\ (\Pi_+^{(p,q)} + \Pi_-^{(p,q)} = 1,\ \Pi_+^{(p,q)}\Pi_-^{(p,q)} = \Pi_-^{(p,q)}\Pi_+^{(p,q)} = 0).\end{gathered} \tag{A2.9}$$

With the help of these operators the irreducible parts of a superfield $\Phi_k(\eta^{\mathrm{R}})$ are expressed as follows:

$$\tilde{\Phi}_k^{\pm}(\eta^{\mathrm{R}}) = (\Pi_{\pm}\Phi(\eta^{\mathrm{R}}))_k = P_k^{\pm} + \bar{\eta}^{\mathrm{R}}(Y^{\pm}N)_k - \tfrac{1}{4}m\bar{\eta}^{\mathrm{R}}\eta^{\mathrm{R}}[1 \pm (1+2q)]P_k^{\pm} \tag{A2.10}$$

where

$$P_k^{\pm} = \mp \frac{m[1 \mp (1+2q)]P_k + 4B_k}{2m(1+2q)} \tag{A2.11}$$

and $Y^{\pm}$ are the operators projecting out of $N_{\alpha k}$ Lorentz irreducible pieces:

$$Y^{\pm} = \pm \frac{1 - \mathrm{i}\gamma_5}{2} \frac{1}{2(1+2q)}[1 \pm (1+2q) + \tfrac{1}{2}\sigma^{\mu\nu}J_{\mu\nu}]. \tag{A2.12}$$

From (A2.10) and (A2.12) it follows that the η^{R}-independent invariant spaces exist only for the minus sign in (A2.10). Indeed, only in this case may we nullify the coefficient for $\bar{\eta}^{\mathrm{R}}\eta^{\mathrm{R}}$ (by putting $q = 0$ there). For the representations $D^{(p,0)}$ the generators $J_{\mu\nu}$ contain the projector $\frac{1}{2}(1 + \mathrm{i}\gamma_5)$; therefore, due to the trivial algebraic property $[\frac{1}{2}(1 - \mathrm{i}\gamma_5)\sigma^{\mu\nu}]_{\alpha}^{\beta}[\frac{1}{2}(1 + \mathrm{i}\gamma_5)\sigma_{\mu\nu}]_{\delta}^{\gamma} = 0$, the component $(Y^- N)_{\alpha k}$ is zero automatically. Analogously, for the supergroup S_+ the η^{L}-independent invariant functions exist provided their external indices belong to the representations $D^{(0,q)}$ of the Lorentz group. These results explain why OSp(1,4) has no left-handed chiral superfields with $q \neq 0$ and right-handed chiral ones with $p \neq 0$ (see § 3.3). The projection operator which singles out the η^{R}-independent invariant spaces of supergroup S_- has the very simple form

$$\Pi_-^{(p,0)} = (2m - \bar{\mathscr{D}}_+^R \mathscr{D}_+^R)/2m. \tag{A2.13}$$

As an example, we list here the simplest irreducible spaces of the supergroup S_-.

(a) *Scalar superfields* ($p = q = 0$)

$$\hat{C}_2^{(0,0)} = 0, \quad \begin{cases} \hat{C}_{1(+)}^{(0,0)} = 2, \tilde{\Phi}^-(\eta^R) = P^+ + \bar{\eta}^R N^+ - \frac{1}{2} m \bar{\eta}^R \eta^R P^+ \\ \hat{C}_{1(-)}^{(0,0)} = 0, \tilde{\Phi}^-(\eta^R) = P^-. \end{cases}$$

(b) *Spinor superfields* ($p = 0$, $q = \frac{1}{2}$ or $p = \frac{1}{2}$, $q = 0$)

$$\hat{C}_2^{(0,1/2)} = 0, \quad \begin{cases} \hat{C}_{1(+)}^{(0,1/2)} = 6, \tilde{\Phi}_\alpha^+(\eta^R) = P_\alpha^+ + N_{\mu\nu}^+ (\sigma^{\mu\nu}\eta^R)_\alpha - \frac{3}{4} m \bar{\eta}^R \eta^R P_\alpha^+ \\ \hat{C}_{1(-)}^{(0,1/2)} = 2, \tilde{\Phi}_\alpha^-(\eta^R) = P_\alpha^- + N^- \eta_\alpha^R + \frac{1}{4} m \bar{\eta}^R \eta^R P_\alpha^- \end{cases} \tag{A2.14}$$

$$\hat{C}_2^{(1/2,0)} = 6, \quad \begin{cases} \hat{C}_{1(+)}^{(1/2,0)} = 5, \tilde{\Phi}_\alpha^{\prime +}(\eta^R) = P_\alpha^{\prime +} + N_\mu^+ (\gamma^\mu \eta^R)_\alpha - \frac{1}{2} m \bar{\eta}^R \eta^R P_\alpha^{\prime +} \\ \hat{C}_{1(-)}^{(1/2,0)} = 3, \tilde{\Phi}_\alpha^{\prime -}(\eta^R) = P_\alpha^{\prime -}. \end{cases} \tag{A2.15}$$

(c) *Vector superfields* ($p = \frac{1}{2}$, $q = \frac{1}{2}$)

$$\hat{C}_2^{(1/2,1/2)} = 6, \quad \begin{cases} \hat{C}_{1(+)}^{(1/2,1/2)} = 9, \tilde{\Phi}_\mu^+(\eta^R) = P_\mu^+ + \bar{N}_\nu^+ (\delta_\mu^\nu - \frac{1}{4}\gamma^\nu\gamma_\mu)\eta^R - \frac{3}{4} m \bar{\eta}^R \eta^R P_\mu^+ \\ \hat{C}_{1(-)}^{(1/2,1/2)} = 5, \tilde{\Phi}_\mu^-(\eta^R) = P_\mu^- + \bar{N}^- \gamma_\mu \eta^R + \frac{1}{4} m \bar{\eta}^R \eta^R P_\mu^-. \end{cases}$$

The boson components of all these superfields have the structure $S + \mathrm{i}P$, $V_\mu + \mathrm{i}A_\mu$, $N_{\mu\nu} + \mathrm{i}N_{\mu\nu}^*$, where S, P, V_μ, A_μ, $N_{\mu\nu}$ and $N_{\mu\nu}^*$ are, respectively, scalar, pseudoscalar, vector, axial vector, antisymmetric tensor and its dual, all real.

Note that for p fixed (hence, $\hat{C}_2$ fixed) there always exist two different values of q to which the same eigenvalue of the Casimir operator $\hat{C}_1$ corresponds. These q are shifted by $\frac{1}{2}$. By the Schur lemma two such representations of the supergroup S_- are equivalent. Thus, the scalar superfield $\tilde{\Phi}^+(\eta^R)$ from the above set is equivalent to the spinor one $\tilde{\Phi}_\alpha^-(\eta^R)$; this may easily be verified by acting on $\tilde{\Phi}^+(\eta^R)$ by the covariant derivative $\mathscr{D}_+^R (P^+ \sim N^-, N_\alpha^+ \sim P_\alpha^-)$. Analogously, one may be convinced that the spinor superfield $\tilde{\Phi}_\alpha^{\prime +}(\eta^R)$ is equivalent to the vector superfield $\tilde{\Phi}_\mu^-(\eta^R)$.

Appendix 3. Generalised Weyl transformations

Let $\phi_k(x)$ be an O(2,3)-covariant field possessing with respect to the O(2,3)-translations the standard transformation properties (2.7):

$$\delta_R \phi_k(x) = -\delta_R x^\mu \partial_\mu \phi_k(x) + \tfrac{1}{2}\mathrm{i} u^{\mu\nu}(\lambda, x)(J_{\mu\nu}\phi(x))_k$$

with $\delta_R x^\mu$ and $u^{\mu\nu}(\lambda, x)$ given by (2.6) and (2.8). We call the generalised Weyl transformations the following family of canonical transformations of the field $\phi_k(x)$:

$$\phi_k(x) \to \tilde{\phi}_k(x) = a^{2n-1+d_\phi}(x)\phi_k(x). \tag{A3.1}$$

Here d_ϕ is the dimensionality of the field $\phi_k(x)$ (in mass units) and the number n is arbitrary (not necessarily integer). It can be verified that the new field $\tilde{\phi}_k$ is transformed under $O(2,3)$-translations by the law

$$\delta_R \tilde{\phi}_k(x) = -\delta_R x^\mu \partial_\mu \tilde{\phi}_k(x) - \tfrac{1}{4}(2n - 1 + d_\phi)\partial_\mu \delta_R x^\mu \tilde{\phi}_k(x) + \tfrac{1}{2}\mathrm{i} u^{\mu\nu}(\lambda, x)(J_{\mu\nu}\tilde{\phi}(x))_k \tag{A3.2}$$

i.e. as the Poincaré-covariant field of conformal weight n, in the conformal group of Minkowski space (with identification $R_\mu = \frac{1}{2}(P_\mu - m^2 K_\mu)$), the canonical form of the conformal transformation arising under the choice $n = \frac{1}{2}$ (in this case the replacement (A3.1) is the standard Weyl transformation from anti-de Sitter to Minkowski space).

For OSp(1,4)-covariant superfields one can define an analogue of the transformation (A3.1) which brings them into the basis where they have standard transformation properties with respect to the Wess–Zumino superconformal group (with identification $Q = (1/\sqrt{2})(S - mT)$). For instance, in the case of scalar chiral superfields $T^{\pm}(x, \theta_{\pm})$ this canonical replacement has the form

$$T^{\pm}(x, \theta_{\pm}) \to \tilde{T}^{\pm(n)}(x, \theta_{\pm}) = [a(x)(1 + \tfrac{1}{2}ma(x)\bar{\theta}_{\pm}\theta_{\pm})]^{2n} T^{\pm}(x, \sqrt{a(x)}\theta_{\pm}) \tag{A3.3}$$

or, in components,

$$\begin{aligned} &A_{\pm}^{(n)}(x) = a^{2n}(x)A_{\pm}(x), \qquad \psi_{\pm}^{(n)}(x) = a^{2n+1/2}(x)\psi_{\pm}(x), \\ &F_{\pm}^{(n)}(x) = a^{2n+1}(x)(F_{\pm}(x) + 2nmA_{\pm}(x)) \end{aligned} \tag{A3.4}$$

(n is the same as in (A3.1)). For the physical components (A3.4) coincides with (A3.1) ($d_A = 1$, $d_\psi = \frac{3}{2}$, $d_F = 2$). In terms of new fields the transformation law (3.12) looks like

$$\begin{aligned} \delta A_{\pm}^{(n)} &= \bar{\beta}'\psi_{\pm}^{(n)}, \\ \delta\psi_{\pm}^{(n)} &= \tfrac{1}{2}(1 \pm i\gamma_5)[(-i\gamma^{\nu}\partial_{\nu}A_{\pm}^{(n)} + F_{\pm}^{(n)})\beta' - inA_{\pm}^{(n)}\gamma^{\mu}\partial_{\mu}\beta'], \\ \delta F_{\pm}^{(n)} &= -i\bar{\beta}'\gamma^{\nu}\partial_{\nu}\psi_{\pm}^{(n)} - i(n - \tfrac{1}{2})\partial_{\mu}\bar{\beta}'\gamma^{\mu}\psi_{\pm}^{(n)} \end{aligned} \tag{A3.5}$$

where $\beta' = (1/\sqrt{2})(1 - imx^{\mu}\gamma_{\mu})\epsilon$. The law (A3.5) coincides with the transformation law of a scalar multiplet of weight n in the subgroup OSp(1,4) of the superconformal group (Wess and Zumino 1974a, b) (the continuation on the whole amount of odd superconformal transformations can be attained by the substitution $\beta' \to \epsilon_1 + ix^{\mu}\gamma_{\mu}\epsilon_2$ in (A3.5), ϵ_1 and ϵ_2 being spinor parameters connected with the generators S and T). Note that the Lagrangian density (4.23) transforms as the F-component of the scalar multiplet of weight $\frac{3}{2}$.

For general superfields the structure of transformations of the type (A3.3) is much more complicated and for this reason we do not give them here. We mention only that components of the Weyl transformed general scalar OSp(1,4)-superfield behave as if they comprise the vector Wess–Zumino multiplet of a certain superconformal weight (any desirable value of the latter can be achieved).

Appendix 4. The derivation of the structure equations

As a first step, we differentiate the decomposition (2.24) and antisymmetrise independent differentials to obtain

$$\begin{aligned} &d_2(G^{-1}\,d_1G) - d_1(G^{-1}\,d_2G) \\ &\quad = i[d_2\mu(d_1) - d_1\mu(d_2)]R + i[d_2\bar{\tau}(d_1) - d_1\bar{\tau}(d_2)]Q + i[d_2\nu(d_1) - d_1\nu(d_2)]M \end{aligned} \tag{A4.1}$$

(for brevity we have suppressed Lorentz indices). Further, the LHS of (A4.1) can be written as the commutator:

$$[G^{-1}\,d_1G, G^{-1}\,d_2G]. \tag{A4.2}$$

Inserting in the latter again the decomposition (2.24), making use of the (anti) commutator algebra (2.1) and finally comparing coefficients of OSp(1,4)-generators in both sides of (A4.1), we arrive at the equations (2.31).

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